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Approximation of Random Homomorphisms and Random Derivations on Banach Algebras via Direct and Fixed Point Methods

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Abstract

In this paper, we prove the approximation of homomorphisms and derivations related to the following functional equation:

$$f(2x+y)+f(2x-y)=f(x+y)+f(x-y)+2f(2x)-2f(x).$$

The results are obtained in random Banach algebras by means of the direct method and the fixed point method.

1 Introduction

Starting with Ulam's question on stability posed in 1940 [13], and following the first response to this question by Hyers [7] in Banach spaces, together with the extension of Hyers' result by Aoki [4] and Rassias [12], the study of stability has become a central topic in mathematics due to its importance and wide range of applications.

With the development of random norms, which are regarded as generalizations of classical norms in normed vector spaces, this concept is used to describe situations in which the exact determination of the norm of an element is not possible because of randomness or uncertainty. This framework is particularly useful in the analysis of vector spaces where traditional norms are inadequate, allowing the study of functions and functional equations in a more flexible and realistic mathematical setting.

The study of Ulam stability in various random spaces has attracted considerable attention from researchers, leading to many valuable results and new conclusions. Several important contributions in this area can be found in the literature (see [1–3, 9, 10, 12]).

In this paper, we study the stability of the additive functional equation

$$f(2x+y)+f(2x-y)=f(x+y)+f(x-y)+2f(2x)-2f(x). \tag{1.1}$$

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Furthermore, we investigate the homomorphism and derivation associated with the above functional equation in random Banach algebras, under the assumption that the function is odd. Using both the direct method and the fixed point method, we obtain several valuable results and meaningful conclusions.

2 Preliminaries

Definition 2.1. [2] A function $T : [0,1] \times [0,1] \to [0,1]$ is called a *triangular norm* (briefly, a *t-norm*) if T satisfies the following conditions:

(a) $T(x,1) = x$ for all $x \in [0,1]$,

(b) $T(x,y) = T(y,x)$,

(c) $T(x,T(y,z)) = T(T(x,y),z)$,

(d) $y \geq z \Rightarrow T(x,y) \geq T(x,z)$.

Definition 2.2. [2] A random normed space (briefly, RN-space) is a triple (X,μ,T), where X is a *vector space*, T is a *continuous t-norm*, and μ is a function from X into D^+ such that the following conditions hold:

(RN_1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$,

(RN_2) $\mu_{\alpha x}(t) = \mu_x\left(\dfrac{t}{|\alpha|}\right)$ for all $x \in X$ and $\alpha \neq 0$,

(RN_3) $\mu_{x+y}(t+s) \geq T\big(\mu_x(t),\, \mu_y(s)\big)$ for all $x, y \in X$ and $t, s \geq 0$.

Definition 2.3. [2] Let (X,μ,T) be an RN-space.

1) A sequence $\{x_n\}$ in X is said to be *convergent* to x in X if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists a positive integer $\mathbb{N}$ such that

$$\mu_{x_n - x}(\varepsilon) > 1 - \lambda \quad \text{whenever } n \geq \mathbb{N}.$$

2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists a positive integer $\mathbb{N}$ such that

$$\mu_{x_n - x_m}(\varepsilon) > 1 - \lambda \quad \text{whenever } n \geq m \geq \mathbb{N}.$$

3) An RN-space (X,μ,T) is said to be *complete* if and only if every Cauchy sequence in X is convergent to a point in X.

A complete RN-space is said to be a *random Banach space*.

Theorem 2.4. *[2] If (X, μ, T) is an RN-space and $\{x_n\}$ is a sequence such that $x_n \to x$, then*

$$\lim_{n\to\infty} \mu_{x_n}(t) \geq \mu_x(t) \quad \textit{almost everywhere.}$$

Definition 2.5. [2] A *random normed algebra* is a random normed space with an algebraic structure such that

$$(\mathrm{RN}_4) \quad \mu_{xy}(ts) \geq \mu_x(t)\mu_y(s) \quad \text{for all } x, y \in X \text{ and all } t, s > 0.$$

Definition 2.6. [8] Let (X, μ, T) and (Y, μ, T) be random normed algebras.

a) An *additive mapping* $H : X \to Y$ is called a *random homomorphism* if

$$H(xy) = H(x)H(y) \quad \text{for all } x, y \in X.$$

b) An *additive mapping* $\delta : X \to Y$ is called a *random derivation* if

$$\delta(xy) = \delta(x)y - \delta(y)x \quad \text{for all } x, y \in X.$$

Definition 2.7. [9] Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if the following conditions hold:

(i) $d(p, q) = 0$ if and only if $p = q$,

(ii) $d(p, q) = d(q, p)$ for all $p, q \in X$,

(iii) $d(p, s) \leq d(p, q) + d(q, s)$ for all $p, q, s \in X$.

Theorem 2.8. *[5] Let (X, d) be a complete generalized metric space, and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then, for each $x \in X$, either*

$$d(J^n x, J^{n+1} x) = +\infty \quad \textit{for all } n \geq 0,$$

or there exists a natural number n_0 such that

(1) $d(J^n x, J^{n+1} x) < +\infty$ for all $n \geq n_0$,

(2) the sequence $\{J^n x\}$ converges to a fixed point y^* of J,

(3) y^* is the unique fixed point of J in the set

$$Y = \{y \in X : d(J^{n_0} x, y) < \infty\},$$

(4)

$$d(y, y^*) \leq \frac{1}{1 - L} d(y, Jy) \quad \textit{for all } y \in Y.$$

Lemma 2.9. *[10] If an odd function $f : X \to Y$ satisfies (1.1) for all $x, y \in X$, then f is additive.*

3 Stability of Functional Equation (1.1) using the Direct Method

Theorem 3.1. *Let X be a real normed algebra, (Y, μ, T) be a random Banach algebra space, and let $f : X \to Y$ be an odd mapping such that $f(0) = 0$ and $f(\lambda x) = \lambda f(x)$ for all $x \in X$ and $\lambda \in \mathbb{R}$. Suppose that the functions $\theta : X^2 \times \mathbb{R} \to D^+$ and $\zeta : X^2 \times \mathbb{R} \to D^+$ satisfy*

$$\mu_{D_f(x,y)}(t) \geq \theta(x, y, t), \tag{3.1}$$

for all $x, y \in X$,

$$\lim_{n\to\infty} \theta(3^n x, 3^n y, 3^n t) = 1, \tag{3.2}$$

$$\lim_{n\to\infty} T_{z=1}^{n} \left(\theta\left(3^{z-1}x, 3^{z-1}y, \frac{t}{3} \right) \right) = 1, \tag{3.3}$$

$$\mu_{f(xy)-f(x)f(y)}(t) \geq \zeta(x, y, t), \tag{3.4}$$

for all $x, y \in X$, and

$$\lim_{n\to\infty} \zeta(3^n x, 3^n y, 3^{2n} t) = 1. \tag{3.5}$$

Then there exists a unique homomorphism $H : X \to Y$ such that

$$\mu_{H(x)-f(x)}(t) \geq T_{z=1}^{n} \left(\theta\left(3^{z-1}x, 3^{z-1}y, \frac{t}{3} \right) \right). \tag{3.6}$$

Proof. If $x = y$, then from (3.1) we have

$$\mu_{D_f(x,x)}(t) \geq \theta(x, x, t).$$

Hence,

$$\mu_{\frac{f(3x)}{3}-f(x)}\left(\frac{t}{3}\right) \geq \theta(x, x, t),$$

and consequently,

$$\mu_{\frac{f(3x)}{3}-f(x)}(t) \geq \theta(x, x, 3t). \tag{3.7}$$

Therefore,

$$\mu_{\frac{f(3^{i+1}x)}{3^{i+1}}-\frac{f(3^i x)}{3^i}}(t) \geq \theta(3^i x, 3^i y, 3^i t),$$

and

$$\mu_{\frac{f(3^{i+1}x)}{3^{i+1}}-\frac{f(3^i x)}{3^i}}\left(\frac{t}{3^{i+1}}\right) \geq \theta\left(3^i x, 3^i y, \frac{t}{3}\right). \tag{3.8}$$

Since

$$\frac{f(3^n x)}{3^n} - f(x) = \sum_{i=0}^{n-1} \left(\frac{f(3^{i+1}x)}{3^{i+1}} - \frac{f(3^i x)}{3^i} \right),$$

and

$$1 > \sum_{i=1}^{n} \frac{t}{3^i} \quad \Rightarrow \quad t > t\sum_{i=1}^{n} \frac{t}{3^i},$$

it follows from the triangle inequality that

$$\begin{aligned}\mu_{\frac{f(3^nx)}{3^n}-f(x)}(t) &\geq \mu_{\frac{f(3^nx)}{3^n}-f(x)}\left(\sum_{i=1}^{n}\frac{t}{3^i}\right)\\ &\geq T_{i=0}^{n-1}\left(\mu_{\frac{f(3^{i+1}x)}{3^{i+1}}-\frac{f(3^ix)}{3^i}}\left(\frac{t}{3^{i+1}}\right)\right)\\ &\geq T_{z=1}^{n}\left(\theta(3^{z-1}x,3^{z-1}x,t)\right).\end{aligned} \tag{3.9}$$

Letting $n\to\infty$ in (3.9), we obtain

$$\mu_{\frac{f(3^nx)}{3^n}-f(x)}(t)=1.$$

Hence, $\left\{\dfrac{f(3^nx)}{3^n}\right\}$ is a Cauchy sequence in (Y,μ,T). Since (Y,μ,T) is complete, the sequence $\left\{\dfrac{f(3^nx)}{3^n}\right\}$ is convergent. Define

$$H(x)=\lim_{n\to\infty}\frac{f(3^nx)}{3^n},\quad \text{for all } x\in X.$$

Clearly, H is an odd mapping.

Replacing x,y with 3^nx and 3^ny, respectively, in (3.1), and then multiplying the right-hand side by $\frac{3^n}{3^n}$, it follows that

$$\mu_{\frac{1}{3^n}D_f(3^nx,3^ny)}(t)\geq\theta(3^nx,3^ny,3^nt),\quad \text{for all } x\in X.$$

Letting $n\to\infty$, we find that H satisfies the functional equation (1.1). Therefore, by Lemma 2.9, H is additive. Moreover, $H(\lambda x)=\lambda H(x)$, and hence H is linear.

To prove that H is a homomorphism, we observe that

$$\begin{aligned}\mu_{H(xy)-H(x)H(y)}(t) &= \mu_{\lim_{n\to\infty}\left(\frac{f(3^{2n}xy)}{3^{2n}}-\frac{f(3^nx)}{3^n}\frac{f(3^ny)}{3^n}\right)}(t)\\ &\geq \lim_{n\to\infty}\zeta(3^nx,3^nx,3^{2n}t)=1.\end{aligned} \tag{3.10}$$

Therefore,

$$H(xy)=H(x)H(y).$$

Finally, assume that there exists another mapping $\widehat{H}:X\to Y$ such that $\widehat{H}$ satisfies the functional equation (1.1) and (3.9). Let

$$\widehat{H}(x)=\lim_{n\to\infty}\frac{g(3^nx)}{3^n}.$$

Then

$$\mu_{\frac{f(3^nx)}{3^n}-\frac{g(3^nx)}{3^n}}(t)\geq T\left(\mu_{\frac{f(3^nx)}{3^n}-f(x)}\left(\frac{t}{2}\right),\mu_{f(x)-\frac{g(3^nx)}{3^n}}\left(\frac{t}{2}\right)\right)$$

$$\geq T\left(T_{z=1}^{n}\left(\theta\left(3^{z-1}x, 3^{z-1}x, \frac{t}{3}\right)\right), T_{z=1}^{n}\left(\theta\left(3^{z-1}x, 3^{z-1}x, \frac{t}{3}\right)\right)\right). \tag{3.11}$$

Letting $n \to \infty$, we obtain

$$\mu_{H(x)-\widehat{H}(x)}(t) = 1,$$

and hence

$$H = \widehat{H}.$$

□

Theorem 3.2. *Let X be a real normed algebra, (Y, μ, T) be a random Banach algebra space, and let $f : X \to Y$ be an odd mapping such that $f(0) = 0$ and $f(\lambda x) = \lambda f(x)$ for all $x \in X$ and $\lambda \in \mathbb{R}$. Suppose that the functions $\theta : X^2 \times \mathbb{R} \to D^+$ and $\zeta : X^2 \times \mathbb{R} \to D^+$ satisfy*

$$\mu_{(D_f(x,y))}(t) \geq \theta(x, y, t), \tag{3.12}$$

for all $x, y \in X$,

$$\lim_{n\to\infty} \theta(3^n x, 3^n y, 3^n t) = 1, \tag{3.13}$$

$$\lim_{n\to\infty} T_{z=1}^{n}\left(\theta\left(3^{z-1}x, 3^{z-1}y, \frac{t}{3}\right)\right) = 1, \tag{3.14}$$

$$\mu_{f(xy)-xf(y)-yf(x)}(t) \geq \zeta(x, y, t), \tag{3.15}$$

for all $x, y \in X$, and

$$\lim_{n\to\infty} \zeta(3^n x, 3^n y, 3^{2n} t) = 1. \tag{3.16}$$

Then there exists a unique derivation $\delta : X \to Y$ such that

$$\mu_{\delta(x)-f(x)}(t) \geq T_{z=1}^{\infty}\left(\theta\left(3^{z-1}x, 3^{z-1}y, \frac{t}{3}\right)\right). \tag{3.17}$$

Proof. The proof follows by applying the same argument used in the proof of Theorem 3.1. □

4 Stability of Functional Equation (1.1) using the Fixed Point Method

Theorem 4.1. *Let X be a real normed algebra and let (Y, μ, T) be a random Banach algebra space. Let $f : X \to Y$ be an odd mapping such that $f(0) = 0$ and $f(\lambda x) = \lambda f(x)$ for all $x \in X$ and $\lambda \in \mathbb{R}$. Suppose there exists a function $\theta : X^2 \times \mathbb{R} \to D^+$ such that*

$$\theta(3x, 3y) \leq 3L\,\theta(x, y), \quad \textit{for some } L < 1,$$

and

$$\mu_{D_f(xy)}(t) \geq \frac{t}{t + \theta(x, y)}, \quad \textit{for all } x, y \in X, \tag{4.1}$$

$$\lim_{n\to\infty} \frac{3^n t}{3^n t + \theta(3^n x, 3^n y)} = 1, \tag{4.2}$$

$$\mu_{f(xy)-f(x)f(y)}(t) \geq \frac{t}{t+\theta(x,y)}, \quad \textit{for all } x, y \in X,\ t > 0. \tag{4.3}$$

Then

$$H(x) = \lim_{n\to\infty} \frac{f(3^n x)}{3^n}$$

exists for each $x \in X$ *and defines a random homomorphism* $H : X \to Y$ *such that*

$$\mu_{f(x)-H(x)}(t) \geq \frac{(3-3L)t}{(3-3L)t + \theta(x,y)}. \tag{4.4}$$

Proof. Setting $x = y$ in (4.1), we obtain

$$\mu_{f(3x)-3f(x)}(t) \geq \frac{t}{t+\theta(x,x)}.$$

Consequently,

$$\mu_{\frac{f(3x)}{3}-f(x)}(t) \geq \frac{3t}{3t+\theta(x,x)}, \quad \text{for all } x \in X,\ t > 0. \tag{4.5}$$

□

Consider the set

$$\mathcal{M} = \{g : X \to Y\}.$$

Define a generalized metric on $\mathcal{M}$ by

$$d(g,h) = \inf\left\{\alpha \in \mathbb{R}^+ : \mu_{g(x)-h(x)}(\alpha t) \geq \frac{3t}{3t+\theta(x,x)},\ \text{for all } x \in X,\ t > 0\right\}.$$

Then $(\mathcal{M}, d)$ is complete (see [5]).

Now, define the linear mapping $J : \mathcal{M} \to \mathcal{M}$ by

$$J(g(x)) = \frac{g(3x)}{3}, \quad \text{for all } x \in X.$$

Let $g, h \in \mathcal{M}$ be such that $d(g,h) = \varepsilon$. Then

$$\mu_{g(x)-h(x)}(\varepsilon t) \geq \frac{3t}{3t+\theta(x,x)}, \quad \text{for all } x \in X,\ t > 0.$$

Moreover,

$$\begin{aligned}
\mu_{Jg(x)-Jh(x)}(L\varepsilon t) &= \mu_{\frac{g(3x)}{3}-\frac{h(3x)}{3}}(L\varepsilon t) \\
&= \mu_{g(3x)-h(3x)}(3L\varepsilon t) \\
&\geq \frac{9Lt}{9Lt+\theta(3x,3x)} \\
&= \frac{3t}{3t+\dfrac{\theta(3x,3x)}{3L}} \\
&\geq \frac{3t}{3t+\theta(x,x)}, \quad \text{for all } x \in X,\ t > 0.
\end{aligned} \tag{4.6}$$

Since $d(g,h)=\varepsilon$ implies that $d(Jg,Jh)\le L\varepsilon$, it follows that

$$d(Jg,Jh)\le L\,d(g,h),\quad \text{for all } g,h\in\mathcal{M}.$$

From (4.5), we obtain

$$d(f,Jf)\le 1.$$

By Theorem 2.8, there exists a mapping $H:X\to Y$ satisfying the following properties:

1. H is a fixed point of J, that is,

   $$J(H(x))=H(x),$$

   which implies

   $$\frac{H(3x)}{3}=H(x),\quad \text{or equivalently,}\quad H(3x)=3H(x).$$

   Hence, for all $x\in X$, the mapping H is the unique fixed point of J in the set

   $$K=\{g\in\mathcal{M}:d(f,g)<\infty\}.$$

   Therefore, there exists $\alpha\in(0,\infty)$ such that

   $$d(f,H)<\alpha,$$

   and consequently,

   $$\mu_{f(x)-H(x)}(\alpha t)\ge\frac{3t}{3t+\theta(x,x)}.$$

2. $d(J^nf,H)\to 0$ as $n\to\infty$, which implies

   $$\lim_{n\to\infty}\frac{f(3^nx)}{3^n}=H(x),\quad \text{for all } x\in X.$$

3. $$d(f,H)\le\frac{1}{1-L}\,d(f,Jf),$$

   which yields the desired inequality

   $$d(f,H)\le\frac{1}{1-L}.$$

This implies that inequality (4.4) holds:

$$\mu_{f(x)-H(x)}(t)\ge\frac{3(1-L)t}{3(1-L)t+\theta(x,x)},\quad \text{for all } x\in X,\ t>0. \tag{4.7}$$

By (4.1), we obtain

$$\mu_{\frac{f(3^n(2x+y))}{3^n}+\frac{f(3^n(2x-y))}{3^n}-\frac{f(3^n(x+y))}{3^n}-\frac{f(3^n(x-y))}{3^n}-\frac{2f(3^n(2x))}{3^n}-\frac{2f(3^n(x))}{3^n}}(t) \geq \frac{3^n t}{3^n t+\theta(3^n x, 3^n y)}, \quad \text{for all } x, y \in X,\ t>0. \tag{4.8}$$

Letting $n \to \infty$, we obtain

$$\mu_{D_H(x,y)}(t) = 1, \quad \text{for all } x, y \in X,\ t>0,$$

that is,

$$\mu_{H(2x+y)+H(2x-y)-H(x-y)-H(x+y)-2H(2x)-2H(x)}(t) = 1.$$

Thus,

$$H(2x+y)+H(2x-y)+H(x+y)+H(x-y)-2H(2x)-2H(x) = 0,$$

which shows that $H : X \to Y$ is additive by Lemma 2.9, and hence H is linear.

Moreover,

$$\mu_{f(xy)-f(x)f(y)}(t) \geq \frac{t}{t+\theta(x,y)}, \quad \text{for all } x, y \in X,\ t>0.$$

Therefore,

$$\mu_{\frac{f(3^{2n}xy)}{3^{2n}}-\frac{f(3^n x)f(3^n y)}{3^{2n}}}(t) \geq \frac{3^n t}{3^n t+\theta(3^n x, 3^n y)}, \quad \text{for all } x, y \in X,\ t>0. \tag{4.9}$$

Since

$$\lim_{n\to\infty} \frac{3^n t}{3^n t+\theta(3^n x, 3^n y)} = 1, \quad \text{for all } x, y \in X,\ t>0,$$

we conclude that

$$\mu_{H(xy)-H(x)H(y)}(t) = 1.$$

Thus,

$$H(xy) - H(x)H(y) = 0,$$

and hence $H(x)$ is a random homomorphism. □

Theorem 4.2. *Let X be a real normed algebra, and (Y, μ, T) be a random Banach algebra space. Let $f : X \to Y$ be an odd mapping with $f(0) = 0$ and $f(\lambda x) = \lambda f(x)$ for all $x \in X$, $\lambda \in \mathbb{R}$. Suppose the function $\theta : X^2 \times \mathbb{R} \to D^+$ satisfies*

$$\theta(3x, 3y) \leq 3L\theta(x,y), \quad L<1,$$

and

$$\mu_{D_f(x,y)}(t) \geq \frac{t}{t+\theta(x,y)}, \quad \textit{for all } x, y \in X, \tag{4.10}$$

$$\lim_{n\to\infty} \frac{3^n t}{3^n t + \theta(3^n x, 3^n y)} = 1, \tag{4.11}$$

$$\mu_{f(xy)-xf(y)yf(x)}(t) \geq \frac{t}{t+\theta(x,y)}, \quad \textit{for all } x, y \in X. \tag{4.12}$$

Then

$$\delta(x) = \lim_{n\to\infty} \frac{f(3^n x)}{3^n}$$

exists for each $x \in X$ *and defines a random derivation* $\delta : X \to Y$ *such that*

$$\mu_{f(x)-\delta(x)}(t) \geq \frac{(3-3L)t}{(3-3L)t+\theta(x,y)}. \tag{4.13}$$

Proof. This theorem can be readily demonstrated by using the same method as in Theorem 4.1. □

Conclusion

In this study, we have obtained results concerning the approximation of homomorphisms and derivations for the functional equation in random Banach algebras. This work represents a new contribution to the study of Ulam–Hyers–Rassias stability and serves as an extension of a series of previous papers authored by the researcher in various other spaces. Furthermore, additional results can be obtained in different spaces and for other types of functional equations, as well as by considering the applied aspects of the study.

References

[1] Alshybani, S., Vaezpour, S. M., & Mahmood, S. J. (2019). Generalized Hyers–Ulam stability of mixed-type additive–quadratic functional equation of random homomorphisms in random normed algebras. *Journal of Physics: Conference Series, 1294*, 032004. https://doi.org/10.1088/1742-6596/1294/3/032004

[2] Al-Shybani, S., Vaezpour, S. M., & Saadati, R. (2018). Generalized Hyers–Ulam stability of a sextic functional equation in random normed space. *Journal of Computational Analysis and Applications, 24*(2), 370–381.

[3] Al-Shybani, S., Vaezpour, S. M., & Saadati, R. (2017). Generalized Hyers–Ulam stability of mixed-type additive–quadratic functional equation in random normed spaces. *Journal of Mathematical Analysis, 5*, 12–25.

[4] Aoki, T. (1950). On the stability of the linear transformation in Banach spaces. *Journal of the Mathematical Society of Japan, 2*, 64–66. https://doi.org/10.2969/jmsj/00210064

[5] Cho, Y. J., Rassias, T. M., & Saadati, R. (2013). *Stability of functional equations in random normed spaces*. Springer. https://doi.org/10.1007/978-1-4614-8477-6

[6] Hsdžić, O., & Pap, E. (2001). *Fixed point theory in probabilistic metric spaces*. Kluwer Academic Publishers. https://doi.org/10.1007/978-94-017-1560-7

[7] Hyers, D. H. (1941). On the stability of the linear functional equation. *Proceedings of the National Academy of Sciences of the United States of America, 27*, 222–224. https://doi.org/10.1073/pnas.27.4.222

[8] Sabah, M. S., & Al-Shybani, S. (2025). Approximation of fuzzy homomorphisms and fuzzy derivations in Banach algebras. *Journal of Al-Qadisiyah for Computer Science and Mathematics, 17*(1), 50–60. https://doi.org/10.29304/jqcsm.2025.17.12009

[9] Sabah, M. S., & Al-Shybani, S. (2025). Approximation of homomorphisms and derivations in Banach algebras via direct and fixed point methods. *Nonlinear Functional Analysis and Applications, 30*(2), 625–646.

[10] Park, C., Lee, J. R., & Shin, D. Y. (2012). Generalized Ulam–Hyers stability of random homomorphisms in random normed algebras associated with the Cauchy functional equation. *Applied Mathematics Letters, 25*(2), 200–205. https://doi.org/10.1016/j.aml.2011.08.018

[11] Park, C., Gordji, M. E., & Saadati, R. (2012). Random homomorphisms and random derivations in random normed algebras via fixed point method. *Journal of Inequalities and Applications, 2012*, 194. https://doi.org/10.1186/1029-242X-2012-194

[12] Rassias, T. M. (1978). On the stability of the linear mapping in Banach spaces. *Proceedings of the American Mathematical Society, 72*, 297–300. https://doi.org/10.1090/S0002-9939-1978-0507327-1

[13] Rheaf, N. D., & Al-Shybani, S. (2024). Some new topological structures in lattice random normed spaces. *Journal of Nonlinear Functional Analysis and Applications*, 1095–1107.

[14] Ulam, S. M. (1960). *A collection of mathematical problems.* Interscience Publishers.