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Predicting the Dynamics of Nigeria's External Reserves Using the Buys-Ballot Method

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Abstract

This study presents a model to predict the dynamics of External Reserves of Nigeria. The choice of suitable time series model, trend estimation and seasonal effect assessment was done using Buy-Ballot method. The result of the analysis showed that an additive model is most suitable for modelling External Reserves of Nigeria between the times considered in the study. The four different trend curves considered for this study which include linear, quadratic, exponential and S-curve were subjected to three test of accuracy such as Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD) and Mean Squared Deviation (MSD) to determine the one with the least error estimate and it was found that the quadratic trend curve had the least error estimates when compared with other trend curves, hence the quadratic trend curve was adopted and fitted to the External Reserves of Nigeria. There was a downward trend as revealed by trend-cycle component which shows that between the periods under study, the External Reserves of Nigeria has been on the decline. The Dot also revealed the influence of seasonal effect on External Reserve of Nigeria. The model equation obtained was used to predict the External Reserve of Nigeria for a period of eight years. The study recommended that the Nigerian government should concentrates more on growing the External Reserves so as to build global community confidence and trust in the nation's fiscal policies and creditworthiness.

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1 Introduction

The surge in oil prices, which had been held in check by careful budgetary management, was primarily responsible for the recent growth in the nation's external reserves. The desire to reduce the financial strain that rising oil prices are having on the economy is driving research into the exploration and use of alternate energy sources. The world's external resources have increased dramatically and quickly in recent years. This remarkable increase illustrates how crucial it is for countries to keep adequate foreign reserves. Production and sales of crude oil constitute almost all of Nigeria's Foreign Exchange Reserves. Its case has changed particularly during 2006 and 2010. The current decline, in spite of rising oil prices, presents questions about the effectiveness of the government's economic management. In order to sustain a desired exchange rate policy, countries have traditionally retained Foreign Exchange Reserves as a tool of significantly manipulating foreign exchange markets. To be able to monitor exchange rates, maintain market stability, and get ready for crises like natural disasters, it is necessary to maintain External Reserves topped off. Although crude oil contributes to more than 90% of Nigeria's Foreign Exchange earnings, the country's capital is susceptible to instabilities in the price of the commodity. These fluctuations have influenced a nation's Foreign Reserves; especially since the year 2000 began. Because of this, the general public is currently debating and worrying about how the reserves should be managed. Scientific explanations for events are crucial for observation as well as for research, forecasting, and acceptance. Scientologists who study theory of science have begun to modify the conventional philosophical theories of scientific explanation in an effort to explain this behaviour.

Time-series forecasting is the process of projecting future events based on historical data. Time-series forecasting models are statistical tools that expect future values by grouping previous data points chronologically. By deducing observed trends and patterns from the data, these models anticipate future values. [1] argues that in order to protect against subsequent financial crises, External Reserves are required. Furthermore, it has been suggested that national reserves serve as a potential asset store utilized by Central Banks to affect the value of their own currency [2–4]. Famous economists including [5–7] have noted in their investigations that Foreign Reserves play a role in the growth of foreign trust in a nation's policies and creditworthiness. A nation's credit rating is raised when it maintains enough reserves, which guarantee that local borrowers may get Foreign Currency to fulfil their obligations for servicing foreign debt [8,9]. But a nation's strong fiscal policies and general investment environment also have a big impact on building trust in the nation [10]. As to [11], the objective of the reserve build-up agenda of Central Banks of Asian was to support their export-driven growth approach by delaying the strengthening of their currencies in regard to the US dollar.

The buys ballot approach, according to [12] can be used to estimate a time series' trend. As a recent breakthrough in statistics, the Buys Ballot approach has proven to be precise as well as successful in identifying the time series trend. $Y = a + bt$ is one way to write the formula. By applying the aforementioned formula, the Time series demonstrates the trend's nature, which is characterized by a pattern in which the periodic mean's plot and the series' overall plot exhibit comparable patterns with regard to; hence, rather than reviewing the series' overall plot. All one has to do is look at the yearly means or the period plot to decide which trend is acceptable. According to [12], Buys Ballot may be employed as well to evaluate the trend and periodic components from the selected descriptive time series model. He concluded by suggesting that the overall mean ($X_{\dots}$) may be utilized for estimating the trend component. The seasonal means ($X_j, j = 1, 2, \dots, m$) and the total mean are used to calculate the seasonal indices. Buys Ballot Tables are utilized to evaluate the effects as a change or a ratio [13]. Put another way, the seasonal influence rises with the width of the deviations, and the relationship that exists amid the regular means and the regular standard deviations, therefore, offers a signal as to the intended model that makes use of the Buys Ballot Table. [14] in their study, revealed that Buys-Ballot estimates for exponential and s-shaped growth curves. The Buys-Ballot estimate approach for temporal disintegration when the trend cycle component is logistical, modified exponential, exponential, or Gompertz has been investigated in this study. Estimates have been determined for the multiplicative and addictive models. [15] studied the Buys-Ballot Modelling of Nigerian local crude oil output using a descriptive methodology. Using the Buys-Ballot approach, the seasonal influence was assessed together with model selection and variance stability (transformation). The findings show the impact of the seasonal effect, with certain seasons (months) considered to be the production's peaks and troughs. [16] conducted research on the application of suitable transformation in time series study of the Nigerian dairy price index (1990-2012). The seasonal effect was assessed using the multiplicative model, variability stability, and model selection in the Buys-Ballot Table technique. The result indicates that there are effect components in the data, as was projected for price data.

In this research, we modelled Nigeria's External Reserves between 2013 and 2022. The model to be hired, the trend estimate, and the seasonal impact of Nigeria's Foreign Reserves were evaluated using the Buys Ballot technique. The model equation obtained was used to predict the External Reserve of Nigeria for a period of eight years (2023-2031).

2 Methodology

The Figures used in this study are monthly External Reserve of Nigeria from 2020-2022. The Figures was obtained from the Central Bank of Nigeria. The data was analysed using the Buys-Ballot method. The method was used to calculate the trend equation and seasonal variation of External Reserves in Nigeria. The method was also used for the assessment of the seasonal component, assessment of trend component, choice of appropriate model and appropriate transformations.

Table 2.1: Buys-Ballot Table

SEASONS (J)								
PERIOD	1	2	j	$\cdots$	S	$T_{i.}$	$\bar{X}_{i.}$	$\sigma_{i.}$
1	X_1	X_2	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\sigma_{1.}$
2	X_{s+1}	X_{s+2}	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\sigma_{2.}$
3	X_{2s+1}	X_{2s+2}	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\sigma_{3.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$X_{(i-1)s+j}$	$\cdots$	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\sigma_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\sigma_{m.}$
$T_{j.}$	$T_{.1}$	$T_{.2}$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{j.}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\sigma_{j.}$	$\sigma_{.1}$	$\sigma_{.2}$	$\sigma_{.j}$	$\cdots$	$\sigma_{.s}$			$\sigma_{..}$

The definition of different parameters and variables in Table 2.1 are stated as follows:

$$\text{Total}(T_j) = \sum_{i=1}^{m} X_{(i-1)s+j}, \quad j = 1, 2, 3, \ldots, s \tag{2.1}$$

$$\text{Total}(T_j) = \text{Total}(T_i) \tag{2.2}$$

$$\begin{aligned}\text{Mean } \bar{X}_j &= \frac{\text{Total}(T_j)}{S}, \quad j = 1, 2, 3, \ldots, s \\ &= \frac{\sum_{i=1}^{m} X_{(i-1)s+j}}{S}\end{aligned} \tag{2.3}$$

$$\begin{aligned}\text{Mean } \bar{X}_i &= \frac{\text{Total}(T_i)}{M}, \quad i = 1, 2, 3, \ldots, m \\ &= \frac{\sum_{j=1}^{s} X_{(i-1)s+j}}{M}\end{aligned} \tag{2.4}$$

$$\textit{Grand Total } (T_{...}) = \sum_{i=1}^{m} T_i = \sum_{j=1}^{s} T_j = \sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)s+j} \tag{2.5}$$

$$\textit{Grand Mean } \bar{X}_{...} = \frac{T_{...}}{N} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)s+j}}{N}, \quad \text{where } N = ms \tag{2.6}$$

$$\textit{Standard Deviation } \hat{\sigma}_i = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(X_{(i-1)s+j} - \bar{X}_i)^2} \tag{2.7}$$

$$\textit{Standard Deviation } \hat{\sigma}_j = \sqrt{\frac{1}{s-1}\sum_{j=1}^{s}(X_{(i-1)s+j} - \bar{X}_j)^2} \tag{2.8}$$

$$\textit{Pooled Standard Deviation } \hat{\sigma}_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{s}\sum_{i=1}^{m}(X_{(i-1)s+j} - \bar{X}_{...})^2} \tag{2.9}$$

where, X_t represent the observed value of the series, the number of period 1 years is m, the periodicity is s while $n = ms$ is the overall number of observations/sample size.

2.1 Validity of Instrument

In any Time series analysis and modelling the first thing to do is to check if there is trend by plotting the original time series with respect to time order. After that other assumption like no outliers, seasonality etc. can be tested. Thus in this study we check if there is trend, outliers and seasonality only with the aid of computer software.

2.1.1 Test for Trend

The result obtained by plotting the original time series using computer software shows that there is a linear trend curve in the data.

2.1.2 Test for Seasonality

The seasonal index is used to measure the performance of each season in the period under study. Thus the plot of periodic standard deviation with respect to time order shows that this is seasonal component in the series.

2.2 Choice of Suitable Times Series Model

The connection between the seasonal mean $\left(\bar{X}_{.j}, j = 1, 2, \ldots, s\right)$ and seasonal deviation give a signal of the suitable time series model. Thus, the scheme of the seasonal means $(\bar{X}_j)$ and the seasonal standard deviation ($\widehat{\sigma}_j$) gives a signal of the preferred model. With this scheme, an additive mode is suitable when the seasonal standard deviations indicate no significant rise or fall comparable to any rise or fall in the seasonal means. In another way, a multiplicative model is suitable when the seasonal standard deviations indicate significant rise or fall comparable to any rise or fall in the seasonal means. Assessment of the plot of the seasonal means and seasonal standard deviation (Figure 4.2) proposed the additive model as the suitable model for external reserves of Nigeria.

2.2.1 Additive Model of Time Series

The outline of additive model of a time series is given by

$$I_t = T_t + S_t + C_t = X_t \tag{2.10}$$

where, T_t is trend component, S_t is seasonal component, C_t is cyclical component, I_t is irregular component.

Subsequently, the Buys-Ballot method is used for short series; it implies that the trend and cycle components are fused together. Consequently, additive model in (2.10) eases to

$$M_t + S_t + I_t = X_t \tag{2.11}$$

where the trend-cycle component is M_t.

3 Measure of Model Accuracy

3.1 Choice of Suitable Trend Curve

To choose a suitable trend curve, three processes of accuracy will be calculated for the linear, exponential, quadratic and s-curve curves to obtain which best fits the time series data. Thus, the trend curve with the least of the three accuracy processes is suitable for the research data.

3.1.1 Mean Absolute Percentage Error (MAPE)

This is a metric that define the accurateness of a forecasting technique. It denotes the average of the absolute percentage error of respective entry in a data set to estimate how precise the forecasted quantities were in contrast with the real quantities. M.A.P.E is often effective for analysing large set of data and requires the use of dataset values other than zero. MAPE is important because it can help an organization develop more accurate forecast for future project. It is computed as follows:

$$MAPE = \sum_{t=1}^{n} \left| \frac{X_t - \bar{X}_t}{X_t} \right| \times 100n, \quad \text{provided } X_t \neq 0 \tag{3.1}$$

where

$$X_t = \text{Variable of interest}$$
$$\bar{X}_t = \text{Fitted value of } X_t$$
$$n = \text{Number of observations.}$$

3.1.2 Mean Absolute Deviation (MAD)

This is the average distance between each data point and the mean, which is the average of the absolute deviations from a central point. It is a summary statistic of dispersion or variability. It is defined as:

$$MAD = \frac{\sum_{t=1}^{n} \left| \frac{X_t - \bar{X}_t}{X_t} \right|}{n} \tag{3.2}$$

where X_t, $\bar{X}_t$, and n are as defined in (3.1).

3.1.3 Mean squared deviation (MSD)

It is computed as follows:

$$MAD = \frac{\sum_{t=1}^{n} \left(\frac{X_t - \bar{X}_t}{X_t} \right)^2}{n} \tag{3.3}$$

3.2 Buys-Ballot Estimates of Parameters of Additive Model

The equation of quadratic trend curve is given by:

$$M_t = a + bt + ct^2, \quad t = 1, 2, 3, \ldots, n \tag{3.4}$$

The relation below gives the estimation of parameters of the trend of the entire series;

$$\bar{X}_i = X_t = a + bt + ct^2 = a^1 + b^1 i + ci^2 \tag{3.5}$$

where

$$a^1 = a - b\left(\frac{s-1}{2}\right) + c\left(\frac{s-1}{2}\right) \tag{3.6}$$

$$b^1 = (bs - cs(s-1)) \tag{3.7}$$

$$c^1 = c(s)^2 \tag{3.8}$$

Here, a^1 is the intercept of quadratic trend curve; curvilinear effect of t on $(\overline{X_i})$ (slope) is b^1.

Applying the estimates in Equations (3.5), (3.6), (3.7) and (3.8) separately, we have the quadratic trend-cycle component as,

$$M_t = a + bt + ct^2 \tag{3.9}$$

where:

$$a = a^1 + b\left(\frac{s-1}{2}\right) - c\left(\frac{s-1}{2}\right)^2 \tag{3.10}$$

$$c = \frac{c^1}{s^1} \tag{3.11}$$

$$b = \frac{b^1 + sc(s-1)}{s} \tag{3.12}$$

Here, a represents the intercept of the quadratic trend curve, b is the slope (i.e. linear effect of t on X_t) and c is the slope coefficient (i.e. curvilinear effect of t on X_t).

3.3 Evaluation of Seasonal Indices

The total mean $(\bar{X})$ and the seasonal mean $(\bar{X}_j)$ of the Buys–Ballot Table are used to evaluate the seasonal effects of the time series. Since the model of External Reserve is additive, we used the variance of the seasonal average to the total averages $(\bar{X}_j - \bar{X}_{...})$ to evaluate the existence or otherwise of seasonality. These are seen as deviations and the wider the deviations, the greater the seasonal effects.

3.3.1 Evaluation of Irregular Component for Additive Time Series Model

Upon obtaining the trend-cycle component, M_t, and the seasonal component, S_t, then the estimation of the irregular component, I_t, can be obtained by rearranging equation (2.11)

$$I_t = X_t - M_t - S_t. \tag{3.13}$$

4 Analysis of Results and Discussion

The first method to time series study is to take a time plot of the original series (Chatfield, 1995). Figure 4.1 shows the graph of the External Reserves of Nigeria (2013-2022)

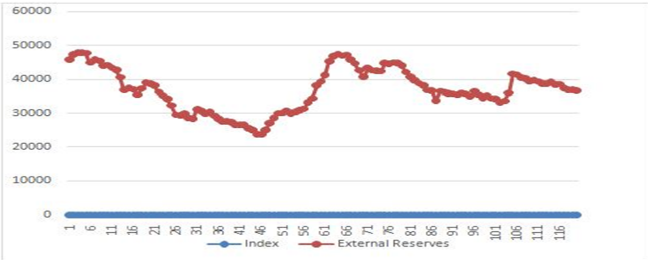

Figure 4.1: Time series plot of External Reserve of Nigeria (2013–2022).

Careful examination of Figure 4.1 shows that External Reserve of Nigeria has fluctuated up and down over the year under investigation. The plot has shown a downfall in External Reserve of Nigeria in recent year.

4.1 Estimation of Parameters of Buys-Ballot Table Using Monthly Reserves of Nigeria

Applying Equations (3.1)-(3.9) to the External Reserves of Nigeria, we obtained Table 4.1.

Table 4.1: Buys-Ballot for External Reserves of Nigeria (2013-2022)

Peroid	1	2	3	4	5	6	7	8	9	10	11	12	Total	Mean i	STD i
1	45,824.44	47,295.85	47,884.12	47,903.09	47,702.88	44,957.00	45,834.11	45,428.84	44,108.48	44,155.11	43,414.20	42,847.31	547,355.43	45,612.95	1784.599
2	40,667.56	36,923.61	37,399.22	37,105.27	35,398.10	37,330.03	39,065.42	38,705.71	38,278.62	36,280.25	35,248.66	34,241.54	446,643.99	37,220.33	1,802.13
3	32,385.71	29,566.99	29,357.21	29,829.75	28,566.54	28,335.21	31,222.81	30,637.17	29,880.21	30,336.36	29,263.02	28,284.82	357,665.80	29,805.48	1213.833
4	27,607.85	27,568.38	27,336.38	26,614.81	26,594.39	26,505.50	25,581.58	25,031.93	23,806.51	23,689.87	25,081.22	26,990.58	312,409.00	26,034.08	1378.06
5	28,592.98	29,975.38	29,996.38	30,749.28	29,811.85	30,340.96	30,898.96	31,278.95	33,159.73	34,323.59	38,207.96	39,347.47	386,683.49	32,223.62	3427.513
6	41,150.28	45,274.00	46,730.54	47,438.22	46,923.01	47,157.90	45,814.20	44,606.79	42,608.95	40,651.23	43,348.25	42,594.84	534,298.21	44,524.85	2401.513
7	42,515.66	42,328.96	44,793.08	44,474.29	44,898.42	44,747.03	43,971.93	42,062.42	40,689.89	39,614.80	38,799.55	38,092.72	506,988.75	42,249.06	2455.196
8	36,730.57	36,599.89	33,689.05	36,459.48	36,203.17	35,779.68	35,559.80	35,511.93	35,964.53	35,580.48	34,938.20	36,476.89	429,493.67	35,791.14	851.5037
9	35,440.88	34,461.46	35,137.84	34,320.73	34,180.21	32,990.72	33,492.40	35,979.76	41,571.37	41,300.11	40,478.08	40,230.80	439,584.36	36,632.03	3263.984
10	39,319.72	39,668.53	39,275.45	38,540.45	38,600.58	39,163.65	38,309.17	38,457.22	37,394.38	36,872.09	36,900.47	36,608.23	459,109.94	38,259.16	1063.566
Total	370,235.65	369,663.05	371,599.27	373,435.37	368,879.15	367,307.68	369,750.38	367,700.72	367,462.67	362,803.89	365,679.61	365,715.20	4,420,232.64		
Mean j	37,0[illegible]57	36,966.31	37,159.93	37,343.54	36,887.92	36,730.77	36,975.04	36,770.07	36,746.27	36,280.39	36,567.96	36,571.52		368,352.72	
STD j	6027.9921	6750.5687	7410.4983	7383.9881	7601.6134	7285.4355	6877.25273	6487.1405	6308.9846	5925.0229	5827.2973	5426.0597			899.2228

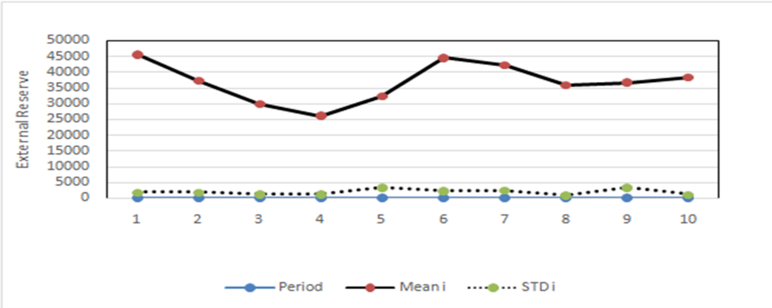

Figure 4.2: Plot of a seasonal mean and seasonal standard deviation.

Figure 4.2 shows that the additive model is suitable for modelling External Reserve of Nigeria. Hence, we identify the best trend curve by applying the three different test accuracy techniques (MAP, MAD and MSD). Statistical packages such as SPSS and EXCEL are used to obtain the test of accuracy values.

Table 4.2: Model summary and parameter estimates For linear model.

Equation	**Model Summary**					**Parameter Estimates**	
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**
Linear	.001	.101	1	118	.751	37157.568	-5.327

The independent variable is INDEpVAR.
Dependent Variable: DEpVAR.
This implies that

$$X_t = 37157.568 - 5.327t \tag{4.1}$$

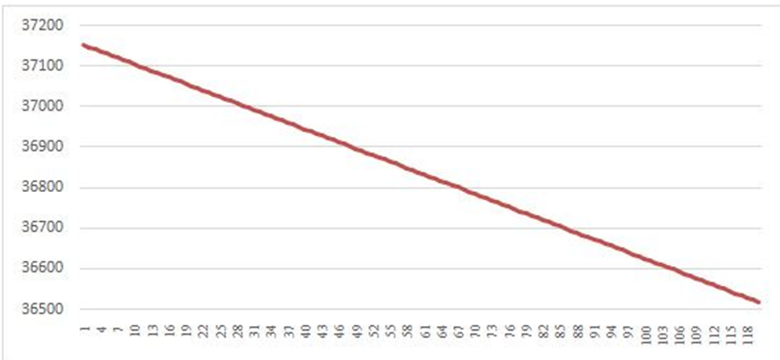

Figure 4.3: Linear trend curve.

Table 4.3: Model summary and parameter estimates For quadratic equation.

Equation	**Model Summary**					**Parameter Estimates**		
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**	**b2**
Quadratic	.001	.053	2	117	.949	37257.548	-10.244	.041

The independent variable is INDEpVAR.
Dependent Variable: DEpVAR.
This implies that

$$X_t = 37257.548 - 10.244t + 0.041t^2 \tag{4.2}$$

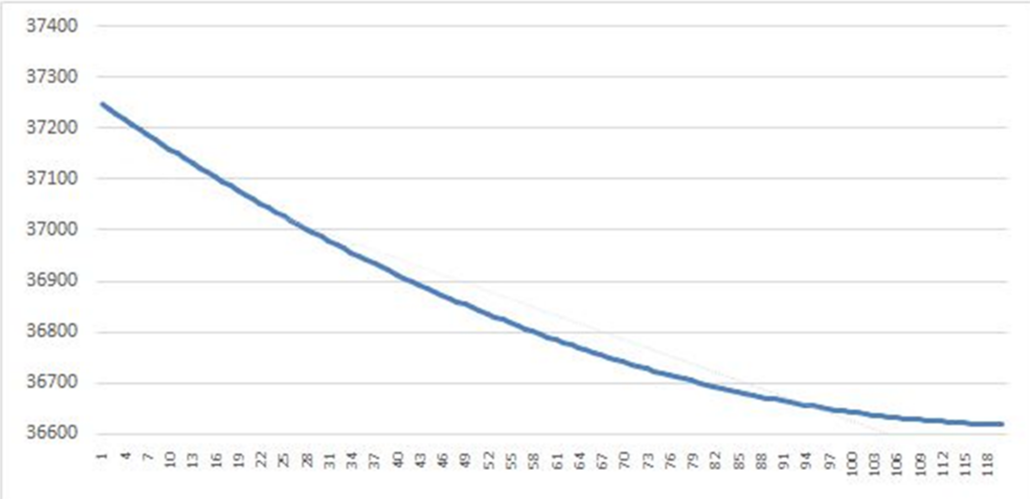

Figure 4.4: Quadratic trend curve.

Table 4.4: Summary of model and parameter estimates For exponential model.

Equation	**Model Summary**					**Parameter Estimates**	
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**
Exponential	.000	.038	1	118	.846	36474.656	-9.169E-005

The independent variable is INDEpVAR.
Dependent Variable: DEpVAR.
This implies that

$$X_t = 36474.656\, e^{-0.00009169} \tag{4.3}$$

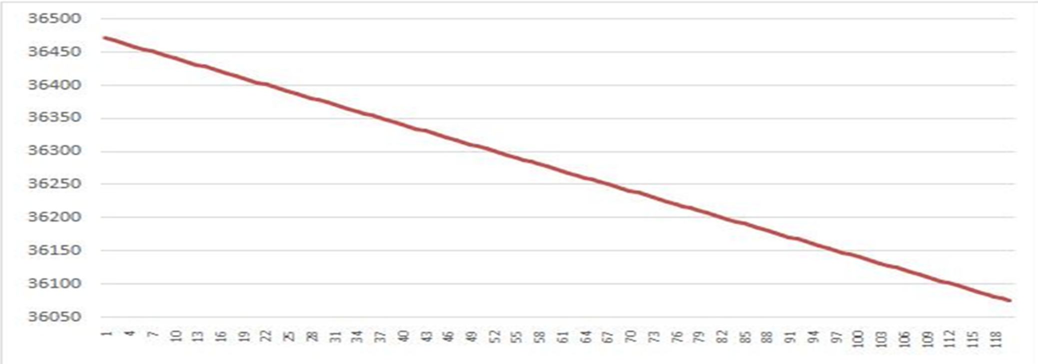

Figure 4.5: Exponential trend curve.

Table 4.5: Summary of model and parameter estimates for S-curve model.

Equation	**Model Summary**					**Parameter Estimates**	
	R Square	**F**	**df1**	**df2**	**Sig.**	**Constant**	**b1**
S	.007	.828	1	118	.365	10.493	.138

The independent variable is INDEpVAR.
Dependent Variable: DEpVAR.
This implies that

$$X_t = 23689.87 + 24213.22\left(\frac{1}{1+e^{-10.493(t-60.5)}}\right) \tag{4.4}$$

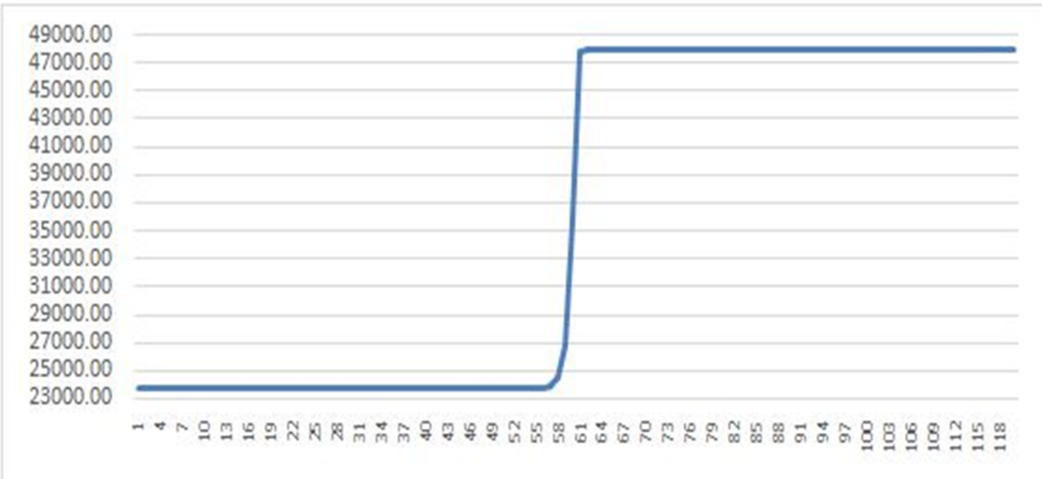

Figure 4.6: S-curve trend.

Table 4.6: Accuracy measure for the best trend curve.

Trend curve	**MAPE**	**MAD**	**MSD**
Linear	15.01	5192.34047	39784038.87
Quadratic	15.00	5182.657133	39782142.87
Exponential	14.86	5221.372384	40104907.19
S-Curve	19.61	12154.6878	189370742.4

In Table 4.6, the value of quadratic, linear, exponential and S-curve trends were compared and it was found that quadratic trend has the least error estimate. Quadratic trend was therefore adopted as the line of best fit for the data on External Reserve of Nigeria. The parameters of the quadratic trend line were therefore estimated.

Applying equations (3.17)-(3.19) to data in Table 4.3 gives

$$\overline{X_i} = 37257.548 + (-10.244) + 0.041t^2 \tag{4.1}$$

Applying Equations (3.21) to (3.23) in (4.1), we obtain the results

$$c = \frac{c^1}{s^2} = \frac{0.041}{12^2} = 0.00024 \tag{4.2}$$

$$b = \frac{b^1 + cs(s-1)}{s} = \frac{-10.244 + (0.00028)(12)(12-1)}{12} = -10.20032 \tag{4.3}$$

$$\begin{aligned} a &= a^1 + b\left(\frac{s-1}{2}\right) - c\left(\frac{s-1}{2}\right)^2 \\ &= 37257 + (-10.20032)\left[\frac{12-1}{2}\right] - 0.041\left[\frac{12-1}{2}\right]^2 \\ &= 37188.968 \end{aligned} \tag{4.4}$$

Substituting (4.2)-(4.4) into (3.15) gives the below (4.5)

$$m_t = a + bt + ct^2 = 37188 - 10.20032t + 0.00028t^2 \tag{4.5}$$

The negative value (-10.20032) in Equation (4.5) indicates that the Nigeria External Reserves is on the decrease because of the linear effect of t on X_t. Replacing the values of $t = 1, 2, 3, .., 120$, we obtain the trend values. Thus Figure 4.7 displays the plot of the de-trended series.

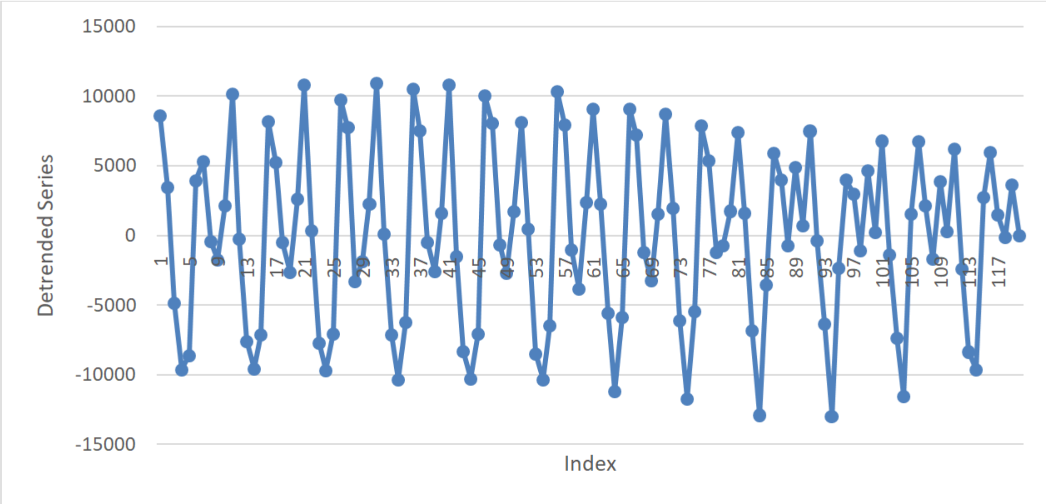

Figure 4.7: Plot of de-trended series.

Careful examination of Figure 4.7 reveals that the trend was deducted from original series, leaving the effects of season and irregular components, and is characterized by decrease in movements.

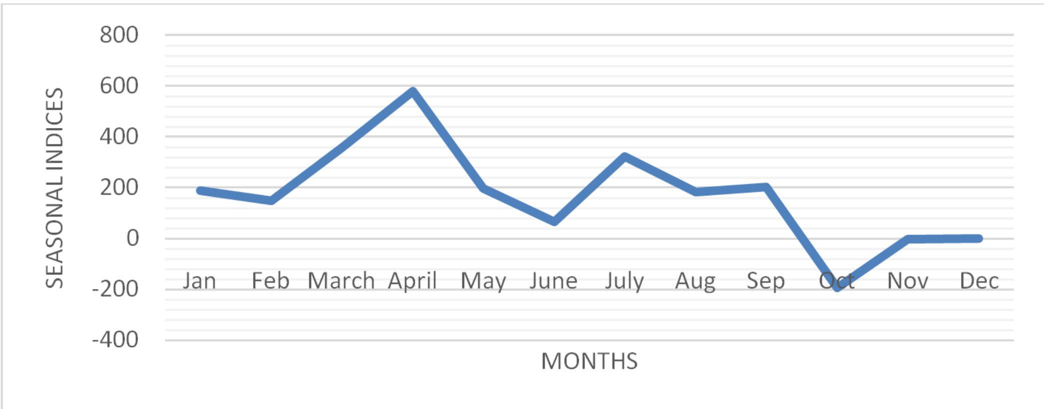

Figure 4.8: Time series of seasonal indices.

Figure 4.8 shows that more External Reserves are recorded during April followed by July and September respectively. Also, the External Reserve appear to be on the decrease during October followed by December, which may be credited to festive times where many domestic products that yield the External Reserve like the crude oil are scarce. We carried out estimation of the seasonal indices by the change in the seasonal averages to the total averages $(\overline{X}_j - \overline{X})$. Figure 4.8 clearly reveals that there is existence of seasonal effects on the series.

4.2 Estimation of Irregular Component for External Reserve of Nigeria

Applying Equation (3.13), we obtain the irregular component of the series. Figure 4.9 shows the graph of the irregular components.

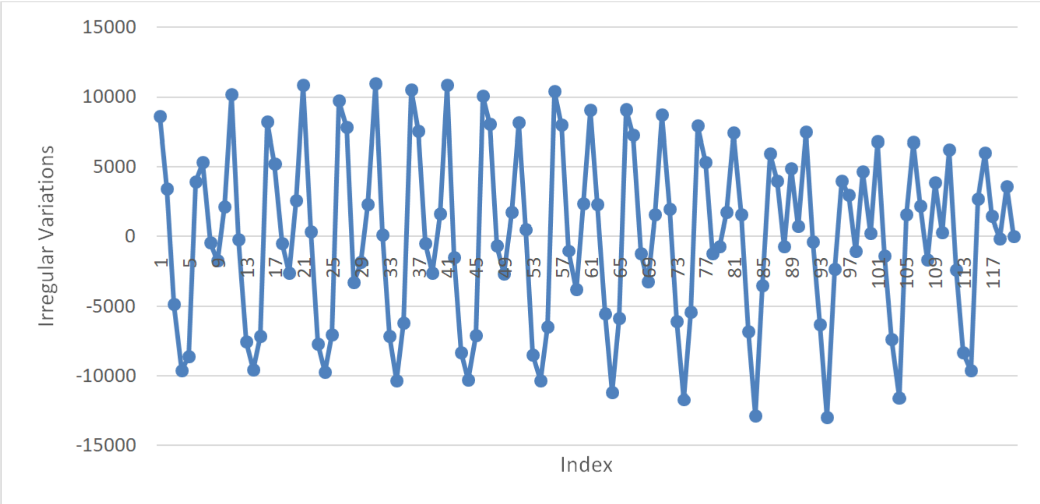

Figure 4.9: Time series plot of irregular components.

4.3 Validation of Normality of Error Component

The application of ordinary least square (OLS) method in estimating the trend parameters in time series analysis often involve the assumption that the error component is random. This may perhaps not always be the case. Therefore, to prove this, we conduct a test on the randomness of the error component using Anderson-Darling test and the result showed that the error component is normal.

Table 4.7: Forecast of External Reserve of Nigeria (2023-2031)

	ESTIMATED			FORCAST		
Actual	**Months**	**Year**	**Estimate**	**Months**	**Year**	**Forecast 2023-2031**
45824.44	1	Jan.2013	37247.35	121	Jan. 2023	36618.31
40667.56	2	Feb.2013	37237.22	122	Feb. 2023	36618.02
32385.71	3	Mar.2013	37227.19	123	Mar. 2023	36617.83
27607.85	4	April. 2013	37217.23	124	April. 2023	36617.71
28592.98	5	May. 2013	37207.35	125	May. 2023	36617.67
41150.28	6	June.2013	37197.56	126	June. 2023	36617.72
42515.66	7	July.2013	37187.85	127	July. 2023	36617.85
36730.57	8	Aug.2013	37178.22	128	Aug. 2023	36618.06
35440.88	9	Sep. 2013	37168.67	129	Sep. 2023	36618.35
39319.72	10	Oct. 2013	37159.21	130	Oct. 2023	36618.73
47295.85	11	Nov.2013	37149.83	131	Nov. 2023	36619.19
36923.61	12	Dec.2013	37140.52	132	Dec. 2023	36619.72
29566.99	13	Jan. 2014	37131.31	133	Jan. 2024	36620.35
27568.38	14	Feb.2014	37122.17	134	Feb. 2024	36621.05
29975.38	15	Mar. 2014	37113.11	135	Mar. 2024	36621.83
45274	16	April. 2014	37104.14	136	April. 2024	36622.70
42328.96	17	May. 2014	37095.25	137	May. 2024	36623.65
36599.89	18	Jun. 2014	37086.44	138	Jun. 2024	36624.68
34461.46	19	July.2014	37077.71	139	July. 2024	36625.79
39668.53	20	Aug. 2014	37069.07	140	Aug. 2024	36626.99
47884.12	21	Sep. 2014	37060.51	141	Sep. 2024	36628.27
37399.22	22	Oct. 2014	37052.02	142	Oct. 2024	36629.62
29357.21	23	Nov. 2014	37043.63	143	Nov. 2024	36631.07
27336.38	24	Dec. 2014	37035.31	144	Dec. 2024	36632.59
29996.38	25	Jan. 2015	37027.07	145	Jan.2025	36634.19
46730.54	26	Feb. 2015	37018.92	146	Jan.2025	36635.88
44793.08	27	Mar. 2015	37010.85	147	Mar. 2025	36637.65
33689.05	28	April. 2015	37002.86	148	April. 2025	36639.50
35137.84	29	May. 2015	36994.95	149	May. 2025	36641.43
39275.45	30	June. 2015	36987.13	150	June. 2025	36643.45
47903.09	31	July. 2015	36979.39	151	July. 2025	36645.55
37105.27	32	Aug. 2015	36971.72	152	Aug. 2025	36647.72
29829.75	33	sep. 2015	36964.15	153	sep. 2025	36649.99
26614.81	34	Oct.2015	36956.65	154	Oct. 2025	36652.33
30749.28	35	Nov. 2015	36949.23	155	Nov. 2025	36654.75
47438.22	36	Dec. 2015	36941.90	156	Dec. 2025	36657.26
44474.29	37	Jan. 2016	36934.65	157	Jan. 2026	36659.85

Actual	Months	Year	Estimate	Months	Year	Forecast 2023-2031
36459.48	38	Feb. 2016	36927.48	158	Feb. 2026	36662.52
34320.73	39	Mar. 2016	36920.39	159	Mar. 2026	36665.27
38540.45	40	April. 2016	36913.39	160	April. 2026	36668.11
47702.88	41	May. 2016	36906.47	161	May. 2026	36671.03
35398.1	42	June. 2016	36899.62	162	June. 2026	36674.02
28566.54	43	July. 2016	36892.87	163	July. 2026	36677.11
26594.39	44	Aug. 2016	36886.19	164	Aug. 2026	36680.27
29811.85	45	sep. 2016	36879.59	165	sep. 2026	36683.51
46923.01	46	Oct.2016	36873.08	166	Oct. 2026	36686.84
44898.42	47	Nov. 2016	36866.65	167	Nov. 2026	36690.25
36203.17	48	Dec. 2016	36860.30	168	Dec. 2026	36693.74
34180.21	49	Jan.2017	36854.03	169	Jan. 2027	36697.31
38600.58	50	Feb.2017	36847.85	170	Feb. 2027	36700.97
44957	51	Mar.2017	36841.75	171	Mar. 2027	36704.71
37330.03	52	April. 2017	36835.72	172	April. 2027	36708.52
28335.21	53	May. 2017	36829.79	173	May. 2027	36712.43
26505.5	54	June. 2017	36823.93	174	June. 2027	36716.41
30340.96	55	July. 2017	36818.15	175	July. 2027	36720.47
47157.9	56	Aug. 2017	36812.46	176	Aug. 2027	36724.62
44747.03	57	Sep. 2017	36806.85	177	Sep. 2027	36728.85
35779.68	58	Oct.2017	36801.32	178	Oct. 2027	36733.16
32990.72	59	Nov. 2017	36795.87	179	Nov. 2027	36737.55
39163.65	60	Dec. 2017	36790.51	180	Dec. 2027	36742.03
45834.11	61	Jan.2018	36785.23	181	Jan. 2028	36746.59
39065.42	62	Feb.2018	36780.02	182	Feb. 2028	36751.22
31222.81	63	Mar.2018	36774.91	183	Mar. 2028	36755.95
25581.58	64	April. 2018	36769.87	184	April. 2028	36760.75
30898.96	65	May. 2018	36764.91	185	May. 2028	36765.63
45814.2	66	June. 2018	36760.04	186	June. 2028	36770.60
43971.93	67	July. 2018	36755.25	187	July. 2028	36775.65
35559.8	68	Aug. 2018	36750.54	188	Aug. 2028	36780.78
33492.4	69	Sep. 2018	36745.91	189	Sep. 2028	36785.99
38309.17	70	Oct.2018	36741.37	190	Oct. 2028	36791.29
45428.84	71	Nov. 2018	36736.91	191	Nov. 2028	36796.67
38705.71	72	Dec. 2018	36732.52	192	Dec. 2028	36802.12
30637.17	73	Jan.2019	36728.23	193	Jan. 2029	36807.67
25031.93	74	Feb.2019	36724.01	194	Feb. 2029	36813.29
31278.95	75	Mar.2019	36719.87	195	Mar. 2029	36818.99
44606.79	76	April. 2019	36715.82	196	April. 2029	36824.78
42062.42	77	May. 2019	36711.85	197	May. 2029	36830.65
35511.93	78	June. 2019	36707.96	198	June. 2029	36836.60
35979.76	79	July. 2019	36704.15	199	July. 2029	36842.63
38457.22	80	Aug. 2019	36700.43	200	Aug. 2029	36848.75

Actual	Months	Year	Estimate	Months	Year	Forecast 2023-2031
44108.48	81	Sep. 2019	36696.79	201	Sep. 2029	36854.95
38278.62	82	Oct.2019	36693.22	202	Oct. 2029	36861.22
29880.21	83	Nov. 2019	36689.75	203	Nov. 2029	36867.59
23806.51	84	Dec. 2019	36686.35	204	Dec. 2029	36874.03
33159.73	85	Jan.2019	36683.03	205	Jan. 2029	36880.55
42608.95	86	Feb.2019	36679.80	206	Feb. 2029	36887.16
40689.89	87	Mar.2019	36676.65	207	Mar. 2029	36893.85
35964.53	88	April. 2019	36673.58	208	April. 2029	36900.62
41571.37	89	May. 2019	36670.59	209	May. 2029	36907.47
37394.38	90	June. 2019	36667.69	210	June. 2029	36914.41
44155.11	91	July. 2019	36664.87	211	July. 2029	36921.43
36280.25	92	Aug. 2019	36662.12	212	Aug. 2029	36928.52
30336.36	93	Sep. 2019	36659.47	213	Sep. 2029	36935.71
23689.87	94	Oct.2019	36656.89	214	Oct. 2029	36942.97
34323.59	95	Nov. 2019	36654.39	215	Nov. 2029	36950.31
40651.23	96	Dec. 2019	36651.98	216	Dec. 2029	36957.74
39614.8	97	JAN.2021	36649.65	217	JAN. 2030	36965.25
35580.48	98	FEB. 201	36647.40	218	FEB. 2030	36972.84
41300.11	99	MAR. 2021	36645.23	219	MAR. 2030	36980.51
36872.09	100	APR. 2021	36643.15	220	APR. 2030	36988.27
43414.2	101	MAY.2021	36641.15	221	MAY. 2030	36996.11
35248.66	102	JUN. 2021	36639.22	222	JUN. 2030	37004.02
29263.02	103	JUL. 2021	36637.39	223	JUL. 2030	37012.03
25081.22	104	AUG. 2021	36635.63	224	AUG. 2030	37020.11
38207.96	105	SEP. 2021	36633.95	225	SEP. 2030	37028.27
43348.25	106	OCT. 2021	36632.36	226	OCT. 2030	37036.52
38799.55	107	NOV. 2021	36630.85	227	NOV. 2030	37044.85
34938.2	108	DEC. 2021	36629.42	228	DEC. 2030	37053.26
40478.08	109	JAN. 2022	36628.07	229	JAN. 2031	37061.75
36900.47	110	FEB. 2022	36626.81	230	FEB. 2031	37070.33
42847.31	111	MAR. 2022	36625.63	231	MAR. 2031	37078.99
34241.54	112	APR. 2022	36624.52	232	APR. 2031	37087.72
28284.82	113	MAY.2022	36623.51	233	MAY. 2031	37096.55
26990.58	114	JUN. 2022	36622.57	234	JUN. 2031	37105.45
39347.47	115	JUL. 2022	36621.71	235	JUL. 2031	37114.43
42594.84	116	AUG. 2022	36620.94	236	AUG. 2031	37123.50
38092.72	117	SEP. 2022	36620.25	237	SEP. 2031	37132.65
36476.89	118	OCT. 2022	36619.64	238	OCT. 2031	37141.88
40230.8	119	NOV. 2022	36619.11	239	NOV. 2031	37151.19
36608.23	120	DEC. 2022	36618.67	240	DEC. 2031	37160.59

5 Conclusion

This study predicts the dynamics of External Reserve of Nigeria. The method of Buy-Ballots was adopted for the analysis and the results showed that the additive model is the most suitable model for modelling External Reserve of Nigeria between the periods under study. In order to obtain the most appropriate trend curve for the study, the four different curves were subjected to three test of accuracy and it was discovered that quadratic trend curve has the least error estimate and was adopted as the most appropriate trend curve for the study. The trend-cycle component showed that the External Reserve of Nigeria has not been stable between the times under review, it fluctuated and this shape was characterized by a parabola. The assessment of the seasonal indices was done using the difference of the seasonal average to the overall average. This was shown in Figure 4.8 which indicates that there is presence of seasonal effects on the series. As anticipated of the data, there is influence of seasonal effects where specific seasons (months) are perceived as the high while some are seen as the low point in the production. The model equation obtained was used to predict the External Reserve of Nigeria from the year 2023 to 2031. It is recommended that the government should concentrate more on growing the External Reserve of Nigeria so as to build global community confidence in the nation's fiscal policies, market stability, creditworthiness, and exchange rate stability.

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