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Sustainable Inventory Models for Raw Material Procurement: Integrating Environmental Resource Management in Manufacturing

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Abstract

Efficient raw material procurement systems play a critical role in advancing environmental sustainability in manufacturing industries. This study presents mathematical inventory models designed to optimize procurement decisions while minimizing environmental impacts such as waste, excessive resource consumption, and energy use. By incorporating deterministic and variable demand scenarios, the models allow manufacturers to align inventory levels with both production needs and sustainable resource use strategies. The raw material procurement inventory model uses two major time elements (periods and circles) to describe procurement and replenishment quantities from several possible sources. Periods are the elapsed time elements between reviews of the stock position while circle is the number of periods occurring between successive procurement actions associated with a sequence of decisions over a period of time in which demand varies. Results from the numerical illustration show that optimized inventory management not only reduces procurement costs but also supports environmentally responsible operations. These findings highlight the synergy between operational efficiency and environmental stewardship in industrial systems.

1 Introduction

The manufacturing sector is a major consumer of natural and environmental resources. Sustainable inventory models in manufacturing industries are increasingly critical as they integrate environmental resource management principles, addressing the pressing need for green supply chain practices within industry frameworks. This is highlighted by recent studies on green supply chain management

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implementation in Ogumeyo et al. [15] which reveal a structured approach essential for enhancing operational efficiency and ecological responsibility. Therefore, sustainable management of raw material inventory is vital in achieving environmentally responsible operations. While inventory is traditionally defined as economic goods kept for future production or sales, the environmental implications of how these inventories are sourced, stored, and used are increasingly important as reported in Pooya et al. [19]. Sustainable inventory models are increasingly recognized as critical frameworks for enhancing the efficiency and environmental performance of raw material procurement in manufacturing industries, emphasizing the integration of environmental resource management to optimize both economic and ecological outcomes (Idisi & Ogumeyo [10]).

By addressing core issues in sustainable supply chain management, mathematical models provide a foundation for developing strategies that balance operational efficiency with ecological sustainability (Seuring & Müller [20]). This study explores inventory models that not only enhance operational efficiency but also contribute to environmental conservation by minimizing waste, reducing carbon footprints, and promoting resource-efficient procurement policies. Economic goods which are kept as inventory could be finished goods (ready for consumption), raw materials needed by manufacturing industries, and work in progress. Qualitative and quantitative questions usually asked in operating an inventory system as contained in Onchoke and Wanyoike [17] and Fred [7] are:

(i) What type of items to order?

(ii) How many of the items to order?

(iii) When to place the order?

(iv) How many of the items do we need to stock in the inventory?

When the stock on hand is maintained at optimum level, there will be an economic advantage from the procurement and inventory process. Hence, the purchasing manager must decide where to procure, how items to procure and when to procure the items to meet customers demand. Ogumeyo et al. [15], states that the objective of inventory control is to make sure industries have the right quantity of stock to meet customers demand without incurring holding costs or out of stock risk in excess. This requires demand forecasting to enables industries align inventory level with anticipated demand level. Techniques such as time series analysis can be used to study the trends and patterns of demand in order to predict anticipated future demand level. Market survey and opinions of experts can also be used to predict the anticipated future demand level. Finally, forecasting of anticipated future demand level in an inventory system can be done through the use of algorithms of mathematical models with the aid of historical data, Ogumeyo and Nwamara [16] and Ogumeyo et al. [15].

According to Oyewobi et al. [18] and Mwangangi [13], the driving force behind the inventory system is **demand**. Demand could be deterministic or stochastic. There are two types of deterministic demand:

- **Static demand**: These are demands which remain constant over a period of time.
- **Dynamic Demand**: These are demands which change over a period of time.

Stochastic Demands: These are demands that are random and governed by probability functions, Silver et al. [21]. Fig. 1 shows a multi-source procurement and inventory scenario in which a procurement manager patronizes three source centers A, B and C for procurement of raw materials in a manufacturing industry.The arrows on the right-hand side denote the inflow of raw materials into the inventory system while the arrows on the left-hand side denote the inflow of finished products into the system. The arrows at the base of Fig. 1 denote demand and replenishing flow.

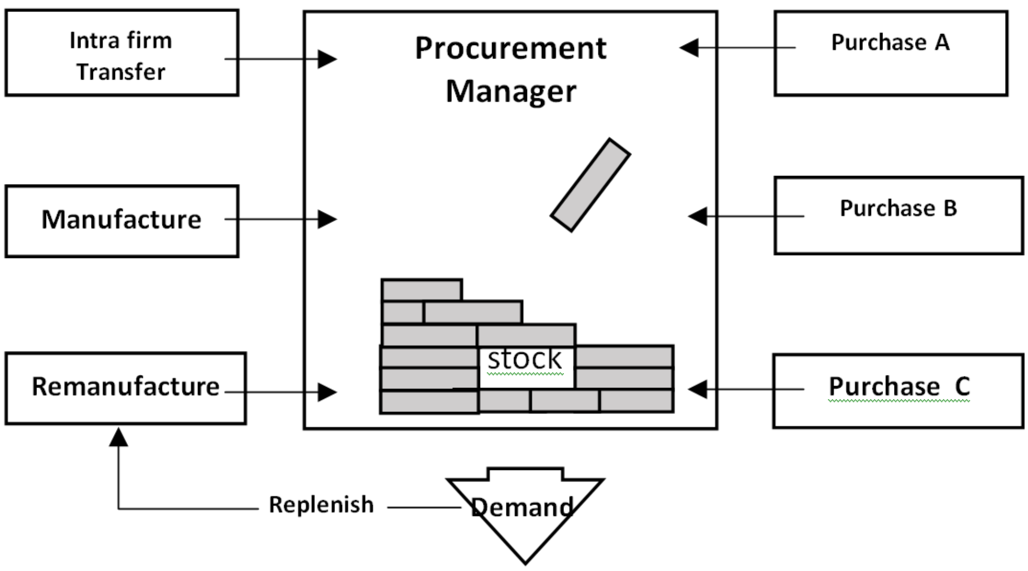

Figure 1: Multi-source procurement and inventory.

Procurement has been defined in Atnafu and Balda [2] as the purchasing of goods and services by either private or public organizations. The study carried out by Auma [3] on logistic challenges in the management of supply chain shows that there is high positive correlation between procurement performance and inventory control in manufacturing industries. According to Hao et al. [9], the performance of procurement can be highly influenced by inventory control management. In many African Countries including Nigeria, much money has been invested in inventory management because of its direct influence on procurement and production of goods and services, Boysen et al. [5] and Mwirigi and Moronge [14]. Inventory control is a process whereby goods and services in stock are regulated in accordance with management inventory policy so as to minimize cost and enhance smooth production process, Achevi et al. [1]. Raw materials' inventory deals with only items needed for manufacturing goods which are used by other production industries. The raw materials are usually stocked in a predetermined limit to ensure smooth operations of those manufacturing industries. Data management is an integral part of inventory system which helps retailers and wholesalers to make better decisions in order to avoid excess inventory.

Inventory management aims at providing the materials required for the sustainability of production operations without interruption in order to satisfy customers' demand. The relationship between inventory management and procurement performance is studied in Mwachiru [12]. The study reveals that inventory management has a high positive impact on procurement performance while the influence of inventory management techniques on the performance of manufacturing firms is studied in Auma [3]. The influence of inventory management on industrial performance using grain bulk handler as a case study is discussed in Mwachiru [12]. Mwangangi [13], studied the influence of inventory management methods on delivering of services using medical supplies in Kenya as a case study. The study shows that effective inventory management minimizes storage cost and wastage.

Procurement process and control of materials being procured is very essential in securing and controlling resources required in an organization, Achevi [1]. Mathematical models on procurement performance and inventory management in literature includes the followings: purchasing and inventory control model discussed in Blackburn [5], procurement performance model using private sectors as a case study is presented in Barasa and Makanzi [4]. A procurement performance and inventory management model on public sector is also discussed in Chimwami et al. [7] while a model which discusses the role of procurement and its performance in public hospitals is developed in Mwirigi and Moronge [14]. Factors which influence management of procurement in public security agencies are explained in Pooya et al. [19]. Ogumeyo and Nwamara [16] studied a decentralized serial supply chain subject to order delays and information distortion, manufacturing and service operation management.

Factors which affect inventory control techniques on service delivery in Nigerian government hospitals are presented in Ogumeyo et al. [15]. A model which uses system dynamics in warehouse management is developed in Cagliano et al. [6] while a framework for government construction and procurement sustainability is presented in Gaur and Bhattacharya [8]. How to achieve procurement sustainability in the face of potential challenges is examined in Kontus [11] and Oyewobi et al. [18]. Atnafu and Balda [2] and Barasa and Mukanzi [4] models on inventory management and procurement of raw materials for manufacturing industries revealed that there is need to maintain adequate stock which could satisfy customers demand at all times in order to control price fluctuation during scarcity in order to minimize wastage. For example, the poor inventory management of pharmaceutical drugs in Tanzania public hospital led to great financial losses, as stated in procurement Audit report in year 2017/2018, contained in Barasa and Mukanzi [4].

Statement of the Problem

The need to evaluate the relationship between inventory management and procurement performance cannot be over emphasized. In manufacturing industries, inventory management policies are put in place to ensure that adequate raw materials are stocked for smooth operations. Poor inventory management can lead to customers dissatisfaction and loss of revenue generation.

Boysen et al. [5] remarked that order and re-order level are determined by several factors such as staff's knowledge of past record, and company's established order and re-order inventory policy. Inappropriate ordered quantities can lead to a situation of either overstocking or under-stocking. As contained in Atnafu and Balda [2], under stock inventory level could be caused by long procurement processes, insufficient fund with which to order for new supplies, lack of trained management staff to regulate the inventory limits and unwillingness of suppliers to supply new order due to delayed payments. Hence, mathematical models which incorporate relevant variables in their formulation are required to guide policy makers in inventory management.

Objectives of the Research

The objective of this study is to develop a mathematical inventory model which shows that:

1. Strategic procurement based on demand forecasts can significantly reduce overstocking, procurement costs, and environmental waste.

2. Balanced procurement timing, minimum holding costs, and replenishment rates, leading to lower energy use and storage needs can be achieved.

3. Integrating sustainability goals into procurement planning contributes to environmental conservation, supports circular economic practices, and enhances manufacturing sustainability.

The model examines the impact of inventory management of economic order quantity on raw material procurement for manufacturing industries. In order to achieve this objective, a model which uses system dynamics in warehouse management in Cagliano et al. [6], a framework for construction and procurement sustainability model presented in Atnafu and Balda [2] and Barasa and Mukanzi [4] model on inventory management and procurement of raw materials for manufacturing industries are being adopted and adjusted in formulating our proposed model.

2 Methodology

Model 1: Raw Material Procurement Inventory with Infinite Replenishment Rate

Model Description and Formulation: The raw material procurement inventory model presented in this section considers a stock of raw materials being maintained to meet the demand of manufacturing industries when the quantity of the raw materials on hand and on order falls to a predetermined level. Two major time elements are involved in procuring a replenishment quantity from one of several possible sources. These are referred to as periods and circles. Periods are the elapsed time elements between reviews of the stock position while circle is the number of periods occurring between successive procurement actions. The review of stock level and adjustment take place at the end of each period. The stock level at the end of one period is equal to the stock level at the starting of the next as shown in Fig. 2.

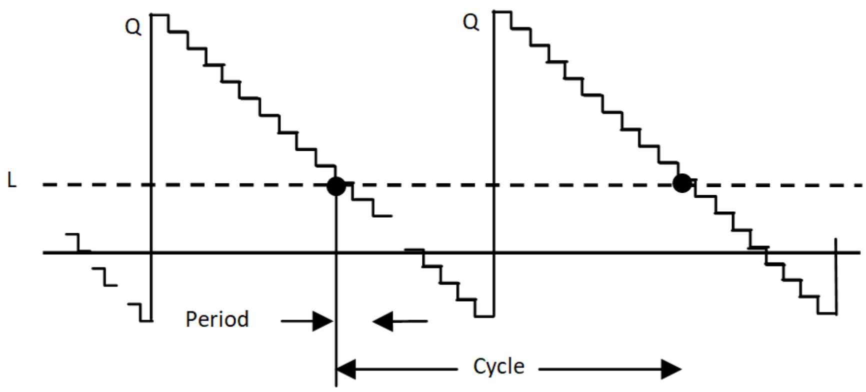

Figure 2: Inventory with instantaneous replenishment.

The objective of the procurement and inventory system is to ensure the demand quantity is supplied at a minimum cost. This can be achieved by assigning appropriate values to decision variables of when to procure, how item to procure and where to procure. This involves constructing a mathematical model of the form:

$$Minimize \quad E = f(x_i, y_j), \tag{1}$$

where E is the measure of effectiveness, x_i represents the policy variables of when to procure and how many items to procure. The y_j is the policy variables denoting replenishment rate, procurement cost, shortage cost and holding cost, Cagliano et al. [6].

Mathematical Symbols

The following symbols are adopted in the model formulation:

TK = Total cost of the system per period

L = Raw material procurement level

Q = Procurement quantity

D = Rate of demand in units per period

T = Lead time in periods

P = Number of periods per cycle

R = Replenishment rate in units per period

K_i = Cost of item per unit

K_p = Procurement cost

K_h = Holding cost per unit per period

K_s = Shortage cost per unit per period

Model Assumptions

The following are the assumptions associated with the raw material procurement model:

(a) The demand for the raw material and the procurement lead time are deterministic.

(b) The rate of raw material replenishment is infinite.

(c) Shortages are not allowed.

The objective of the model is to determine the procurement level, the quantity of the raw material to be procured, and the procurement source in order to minimize the sum of all costs associated with the procurement and the inventory system, in line with Atnafu and Balda [2] model. Hence, the total inventory cost per period is the sum of the raw material cost for the period, the procurement cost of the period, and the holding cost of the period. That is

$$TK = IK + PK + HK. \tag{2}$$

From the model description, the raw material cost for the period will be the item cost per unit times the demand rate in units per period. That is

$$IK = K_i D. \tag{3}$$

On the other hand, the procurement cost for the period will be the purchase cost per purchase divided by the number of periods per cycle. This can be written as

$$PK = \frac{K_p}{P}. \tag{4}$$

But since P is the procurement quantity divided by the demand rate,

$$PK = \frac{K_p D}{Q}. \tag{5}$$

The average inventory during the period will be $Q/2$ since the maximum number of units in stock is Q.

Hence, the holding cost for the period will be the holding cost per unit times the average number of units on hand for the period. This can be stated as

$$HK = \frac{K_h Q}{2}. \tag{6}$$

The total system cost per period will be the sum of the raw material cost per period + the procurement cost per period + and the holding cost per period. This can be mathematically expressed as

$$TK = K_i D + \frac{K_p D}{Q} + \frac{K_h Q}{2}. \tag{7}$$

In order to obtain the procurement quantity which will minimize the total system cost, we differentiate Equation (7) with respect to Q, which is the procurement quantity, and equate the result to zero. Then, we solve for Q as stated in Equation (8).

$$\frac{dTK}{dQ} = -\frac{K_p D}{Q^2} + \frac{K_h}{2} = 0. \tag{8}$$

By simplifying Equation (8) and solving for the value of Q, we have Equation (9).

$$Q = \sqrt{\frac{2K_p D}{K_h}}. \tag{9}$$

Following the procedure of the model assumption stated above, no shortages were allowed, since it was assumed that shortage cost was infinite, in line with Barasa and Mukanza [4]. Hence, the procurement level can be written as

$$L = DT. \tag{10}$$

Therefore, the minimum total system cost can be obtained by substituting the minimum procurement cost quantity into Equation (7), which is the total system cost, to get Equation (11).

$$\text{Minimize } TK = K_i D + \frac{K_p D}{\sqrt{2K_p D/K_h}} + \frac{K_h\sqrt{2K_p/K_h}}{2}. \tag{11}$$

By simplifying Equation (10), we have Equation (12) as follows:

$$MinTK = K_i D + \frac{K_p D}{2} + \sqrt{2K_p K_h D}. \tag{12}$$

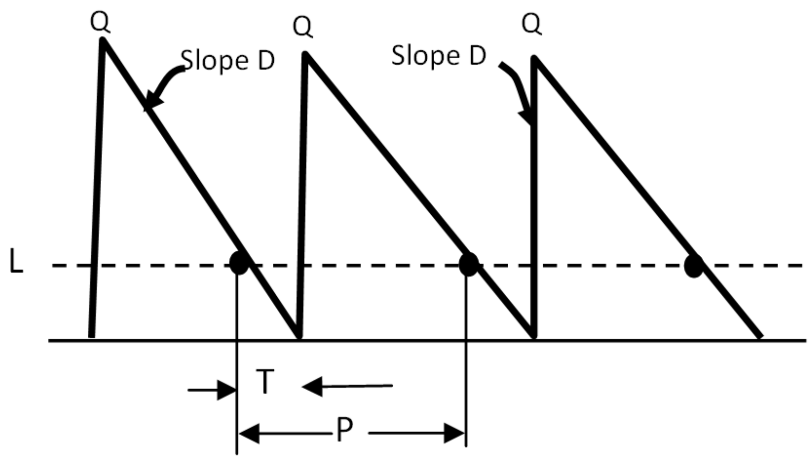

Figure 3: Infinite replenishment rate with infinite shortage cost (Achevi [1]).

Model 2: Raw Material Inventory Model with Finite Replenishment Rate

Model Description and Formulation

If manufacturing the items is to be done once a year in order to meet up with demand, then the cost of initiating production operation will take place once in a year. In this case, there will be high inventory holding cost for the year. On the other hand, if manufacturing operation is done many times per year, the cost of manufacturing will also be incurred many times in a year. Although the holding inventory cost will be relatively small because of the small quantities being produced per time. Hence, the procurement quantity and procurement level must be done in such away so that the total cost of inventory is minimized. Minimum cost of manufacturing items is determined by following the same procedure applied in model 1 for determining the minimum cost of procurement of raw materials. The only difference is that in model 1, the quantity produced is receive at one time while in model 2, the items produced accumulate overtime and the rate of replenishment is finite.

Model Assumptions

The model has the following assumptions:

(i) The demand for the raw material is deterministic.

(ii) The replenishment rate is finite ($R > D$).

(iii) The procurement lead time is deterministic.

Model Formulation

Similar to the procedure adopted in model 1, the total cost of the raw material inventory is the sum of the cost of the item for the period, the procurement cost and the holding cost of the period. This is obtainable by applying Equations (2) to (7). So that the maximum number of units in stock can be expressed as

$$\text{Max } I = t(R - D) = \frac{Q}{R}(R - D). \tag{13}$$

Hence, the average number of units on hand during the period will be the maximum number of units in stock divided by 2 and the holding cost for the period times the average number of units will be the holding cost per period on hand for the period. This is illustrated in Fig. 4.

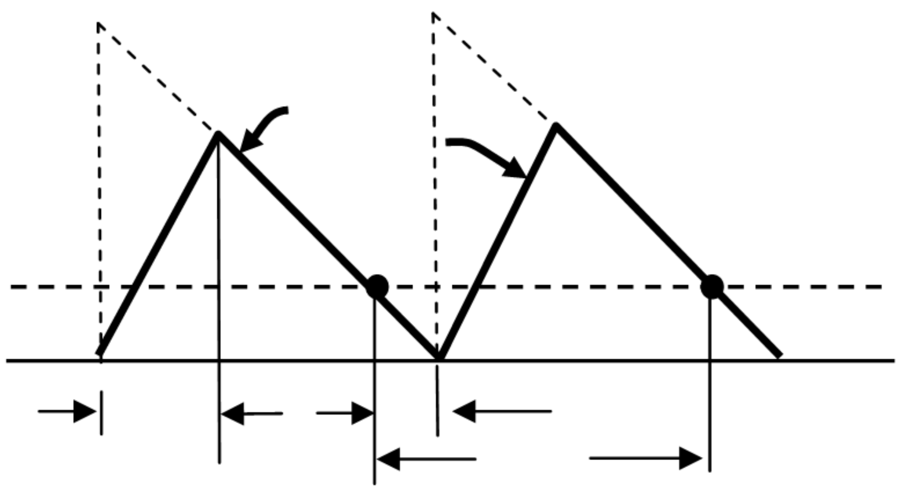

Figure 4: Finite replenishment rate with infinite shortage cost.

That is,

$$TK = \frac{K_h Q}{2R}(R - D). \tag{14}$$

The total system cost per period will be the sum of the raw materials cost per period + the procurement cost per period + the holding cost per period as expressed in Equation (15).

$$TK = K_i D + \frac{K_p D}{Q} + \frac{K_h Q}{2R}(R - D). \tag{15}$$

The quantity procured which results in a minimum total system cost can be obtained by differentiating Equation (15) with respect to Q which is the ordered quantity. If we set the result equal to zero, and solve for Q we obtain the following expressions:

$$\frac{dTK}{dQ} = \frac{K_p D}{Q^2} - \frac{C_h}{2R}(R - D) = 0, \tag{16}$$

$$\therefore Q = \sqrt{\frac{2K_p D}{K_h\left[1 - (D/R)\right]}}. \tag{17}$$

Since shortages are not allowed based on the model assumptions, the minimum total cost of the system can now be obtained if we substitute Equation (17) into the minimum procurement cost in Equation (15). Thus, we have Equation (18).

$$TK_{\min} = K_t D + \frac{K_p D}{\sqrt{2K_p D/K_h\left[1 - (D/R)\right]}} + K_p\sqrt{2K_p D/K_h\left[1 - (D/R)\right](R - D)}. \tag{18}$$

If we simplify Equation (18), we have

$$TK_{\min} = K_t D + \sqrt{2K_p D/K_h\left(1 - \frac{D}{R}\right)K_h D}. \tag{19}$$

In line with Ogumeyo and Nwamara [16], Equation (16) is reduced to Equation (9) and Equation (18) is reduced to Equation (12) as R (the replenishment rate) tends to infinity. Hence, model 2 is a special version of the manufacturer's decision model presented in model 1.

A model of procurement for a sequence of decisions over a period of time is presented in the next section. Dynamic demands are demands which change over a period of time. These occur when demands for raw materials in manufacturing industries varies with time. In such cases, it is assumed that the quantity of raw materials to be procured in order to produce the quantity of items which meet customers' demand must be predetermined at the starting of each time period. Hence the procurement cost and holding cost must be specified. It is also assumed that demand occurs at the starting of the period and that shortages are not allowed.

Model 3: Variable Demand and Procurement

Let I_i be the stock level for period i, let Q be the procured quantity at the start of the period, and let D_i be the demand at the start of the period. Let $(PK)_i$ denote the cost of procurement for the period and $(HK)_i$ the holding cost. Let M be the maximum capacity of the warehouse. The objective of the inventory model is to find the minimum total cost of the system over a given period of time in line with Achevi [1]. This can be expressed as:

$$TK = \sum_{i=1}^{n} \left[(PK)_i + (HK)_i\right]. \tag{20}$$

The decision to procure a quantity Q of raw materials at the start of the period will make the stock level rise up to I_i in that period. This means:

$$L_{i-1} = I_i + D_i - Q_i. \tag{21}$$

Since the stock level cannot exceed the maximum capacity of the warehouse M or be less than zero units, we have the following expression:

$$0 \leq I_{i-1} \leq M. \tag{22}$$

If we substitute the value of I_{i-1} in Equation (21) into Equation (22), we obtain:

$$0 \leq I_i + D_i - Q_i \leq M. \tag{23}$$

By solving for Q, we have:

$$I_i + D_i \geq Q_i \geq I_i + D_i - M. \tag{24}$$

By applying the principle of Bellman optimality in dynamic programming discussed in Pooya et al. [19], Equation (24) yields the following computational equation which we can use to evaluate the minimum procurement total cost over a given planning horizon. That is

$$f_N(I_N) = \min_{I_N+D_N \geq I_N+D_{N-1}-M} \left\{g_N(Q_N + I_N) + f_{N-1}(I_{N-1})\right\}. \tag{25}$$

Substituting for I_{N-1} in Equation (21) gives:

$$f_N(I_N) = \min_{I_N+D_N \geq I_N+D_{N-M}} \left\{g_N(Q_N + I_N) + f_{N-1}(I_N + D_N - Q_N)\right\}. \tag{26}$$

Equation (26) is the procurement minimum total cost of the Nth stage while the computation for the first-stage minimum total procurement cost is given in Equation (27) in line with Oyewobi et al. [18] and Mwangangi [13] for dynamic demand and procurement of raw material models where demand changes over a period of time.

$$f_1(I_1) = \text{Minimize}\left\{g_1(Q_1 + I_1)\right\}. \tag{27}$$

4. Numerical Illustrations

A manufacturing industry, XYZ Limited company demand's for raw materials to produce a certain commodity is to be spread into five-month planning horizon with a requirement schedule of 1000, 2000, 3000, 3000 and 2000 units. The procurement quantities must be in increments of 1000 units with a maximum quantity of 6000 units. The cost of procurement per unit is ₦2000. The inventory holding cost for each period is based on the stock level at the end of the period and is ₦100 per unit per period. Given that the maximum stock level is 4000 units. Determine the procurement quantities over the five-month planning horizon that will minimize the total procurement cost of the system.

Solution

The algorithm discussed in Model 3 can be applied to the numerical raw material procurement inventory problem presented in Section 4 as follows.

Computational Procedure

We start by computing the cost of the first stage from Equation (27) as follows:

$$f_1(I_1) = \min\{g_1(Q + I_1)\}.$$

For values of I_2 ranging from 0 to 4000 units and $D_1 = 1000$, this gives

$$f_1(0) = g_1(1000 + 0) = 2000 + 0 = 2000$$

$$f_1(1000) = g_1(2000 + 1000) = 2000 + 100 = 2100$$

$$f_1(2000) = g_1(3000 + 2000) = 2000 + 200 = 2200$$

$$f_1(3000) = g_1(4000 + 3000) = 2000 + 300 = 2300$$

$$f_1(4000) = g_1(5000 + 4000) = 2000 + 400 = 2400$$

This completes the computation of $f_1(I_1)$. Each value is entered in the first stage of Table 1 together with its associated value of Q_1. The second-period values, $f_2(I_2)$, can be computed using the result of $f_1(I_1)$.

Thus, for the second period, we have:

$$f_2(I_2) = \min_{I_2+D_1 \geq Q_1 \geq I_2+D_2-M} \{g_2(Q_2+I_2) + f_1(I_2+D_2-Q_2)\}.$$

When $I_2 = 0$,

$$f_2(0) = \min_{0 \leq Q_1 \leq 40} \{g_2(Q_2+0) + f_1(0+2000-Q_2)\}.$$

For values of Q_2 ranging from 0 to 2000, this gives

$$f_2(0) = \min \begin{cases} g_2(0+0) + f_1(0+2000-0) = 0+0+2200 = 2200, \\ g_2(10+0) + f_1(0+2000-1000) = 2000+0+2100 = 4100, \\ g_2(20+0) + f_1(0+2000-2000) = 2000+0+2000 = 4000. \end{cases}$$

When $I_2 = 1000$

$$f_2(1000) = \min_{0 \leq Q_2 \leq 3000} \{g_2(Q_2+1000) + f_1(1000+2000-Q_2)\}.$$

For values of Q_2 ranging from 0 to 3000, this gives

$$f_2(10) = \min \begin{cases} g_2(0+1000) + f_1(1000+2000-0) = 0+100+2300 = 2400, \\ g_2(1000+1000) + f_1(1000+2000-1000) = 2000+100+2200 = 4300, \\ g_2(2000+1000) + f_1(1000+2000-2000) = 2000+100+2100 = 4200, \\ g_2(3000+1000) + f_1(1000+2000-3000) = 2000+100+2000 = 4100. \end{cases}$$

When $I_2 = 2000$

$$f_2(2000) = \min_{0 \leq Q_2 \leq 4000} \{g_2(Q_2+2000) + f_1(2000+2000-Q_2)\}.$$

For values of Q_2 ranging from 0 to 4000, this gives

$$f_2(20) = \min \begin{cases} g_2(0+2000) + f_1(2000+2000-0) = 0+200+2400 = 2600, \\ g_2(1000+2000) + f_1(2000+2000-1000) = 2000+200+2300 = 4500, \\ g_2(2000+2000) + f_1(2000+2000-2000) = 2000+200+2200 = 4400, \\ g_2(3000+2000) + f_1(2000+2000-3000) = 2000+200+2100 = 4300, \\ g_2(4000+2000) + f_1(2000+2000-4000) = 2000+200+2000 = 4200. \end{cases}$$

When $I_2 = 3000$

$$f_2(3000) = \min_{1000 \leq Q_2 \leq 5000} \{g_2(Q_2+3000) + f_1(3000+2000-Q_2)\}.$$

For values of Q_2 ranging from 1000 to 5000, this gives

$$f_2(30) = \min \begin{cases} g_2(1000+3000) + f_1(3000+2000-1000) = 2000+300+2400 = 4700, \\ g_2(2000+3000) + f_1(3000+2000-2000) = 2000+300+2300 = 4600, \\ g_2(3000+3000) + f_1(3000+2000-3000) = 2000+300+2200 = 4500, \\ g_2(4000+3000) + f_1(3000+2000-4000) = 2000+300+2100 = 4400, \\ g_2(5000+3000) + f_1(3000+2000-5000) = 2000+300+2000 = 4300. \end{cases}$$

When $I_2 = 4000$

$$f_2(4000) = \min_{2000 \le Q_2 \le 6000} \{g_2(Q_2+4000) + f_1(4000+2000-Q_2)\}.$$

For values of Q_2 ranging from 2000 to 6000, this gives

$$f_2(40) = \min \begin{cases} g_2(2000+4000) + f_1(4000+2000-2000) = 2000+400+2400 = 4800, \\ g_2(3000+4000) + f_1(4000+2000-3000) = 2000+400+2300 = 4700, \\ g_2(4000+4000) + f_1(4000+2000-4000) = 2000+400+2200 = 4600, \\ g_2(5000+4000) + f_1(4000+2000-5000) = 2000+400+2100 = 4500, \\ g_2(6000+4000) + f_1(4000+2000-6000) = 2000+400+2000 = 4400. \end{cases}$$

This completes the computation of $f_2(I_2)$ in the fifth column of Table 1. The minimum value of $f_2(I_2)$ is identified for each value of I_2 and entered in the second stage of Table 1. The values of $f_3(I_3)$, $f_4(I_4)$, and $f_5(I_5)$ are calculated in the same manner, and the minimum results are used to complete Table 1 as follows:

Table 1: Variable Demand and Procurement for the Five-Month Planning Horizon

I	Q_1	$f_1(I)$	Q_2	$f_2(I)$	Q_3	$f_3(I)$	Q_4	$f_4(I)$	Q_5	$f_5(I)$
0	1000	2000	0*	2200	3000*	4200	0	4500	0*	6400
1000	2000	2100	0	2400	4000	4300	0	4900	3000	6600
2000	3000*	2200	0	2600	5000	4400	5000*	4900	4000	6700
3000	4000	2300	5000	4300	6000	4500	6000	6400	5000	6800
4000	5000	2400	6000	4400	6000	4800	6000	6500	6000	6900

5. Analysis of Results

The minimum values of $f_1(I_1)$, $f_2(I_2)$, $f_3(I_3)$, $f_4(I_4)$ and $f_5(I_5)$ identified after using the computational procedure outlined in model 3 are stated in Table 1. Thus, the minimum of the total system cost for the five-month planning horizon is observed to be ₦64,000. This is associated with Q_5 (quantities procured in the fifth period) which has zero units. From Equation (21), the value of I_4 is found to be $0+2000-0=2000$ units with reference to Table 1. This implies that Q_4 (quantities procured in the fourth period) should be 5,000 units. Again, applying Equation (21) gives a value for I_3 of $2000+3000-5000=0$. This gives a value for Q_3 (quantities procured in the third period), which is 3,000 units as shown in Table 1. By following the same procedure of the analysis, we obtain $Q_2=0$ and $Q_1=3000$ units for the quantities procured in periods 2 and 1, respectively. The asterisks in Table 1 denote the vector of procured minimum quantities which were previously identified. Thus, in period 2 and period 5 we have zero procurement, and their minimum total procurement costs for $f_2(I_2)$ and $f_5(I_5)$ are ₦2200 and ₦6400, respectively. This implies that there should be no procurement in the second and fifth periods, while in periods 1 and 3, we should procure 3,000 units each at a minimum cost of ₦2200 for $f_1(I_1)$ and ₦4200 for $f_3(I_3)$, respectively. In period 4, procure 5,000 units of the raw materials at a minimum total cost of ₦4500. The results from the dynamic programming application to the variable demand procurement model reveal a well-optimized procurement strategy over a five-month horizon for a manufacturing firm. The strategy produced a total minimum procurement cost of **₦64,000**, as shown in Table 1, with strategic zero-procurement in the second and fifth months. This aligns with the principle of sustainable procurement, where resource utilization is balanced against demand fluctuations to minimize environmental and financial waste.

A closer inspection of the optimal procurement schedule shows that:

- **3000 units** should be procured in both the **first** and **third periods**,
- **5000 units** in the **fourth period**, and
- **no procurement** in the **second** and **fifth periods**.

This approach not only satisfies fluctuating demand but also minimizes excess stock accumulation and unnecessary ordering — a critical factor in sustainable inventory practices. The computational model reflects the real-world advantage of minimizing environmental footprints through smart procurement timing, which can reduce storage/energy costs, material obsolescence, and transportation-related emissions. Moreover, integrating this procurement plan with environmental resource management practices implies a significant reduction in overproduction, raw material spoilage, and energy consumption — all of which are key concerns in sustainable manufacturing systems. By avoiding surplus inventory, manufacturers can significantly cut down on the carbon footprint associated with storage and logistics, aligning with global sustainability goals (e.g., SDG 12 – Responsible Consumption and Production). The model also illustrates that aligning inventory policies with demand forecasts and procurement timing leads to optimized use of resources and improved operational resilience. This aligns with previous findings by Mwangangi [13] and Achevi [1], who emphasize the importance of data-driven inventory controls in reducing environmental and economic losses.

6. Conclusion

In this study, we have developed and analyzed three procurement inventory models: Raw Material Procurement Inventory Model with infinite replenishment rate, Raw Material Procurement Inventory Model with finite replenishment rate, and Variable Demand Procurement Models within the framework of sustainable industrial resource management. The findings confirm that optimized inventory systems are not only economically efficient but also critical to reducing the environmental impacts of manufacturing operations.

Through mathematical modeling and scenario analysis, the study demonstrates that:

1. Strategic procurement based on demand forecasts can significantly reduce overstocking, procurement costs, and environmental waste.

2. Dynamic decision models help balance procurement timing, holding costs, and replenishment rates, leading to lower energy use and storage needs.

3. Integrating sustainability goals into procurement planning contributes to environmental conservation, supports circular economic practices, and enhances manufacturing sustainability.

The computational illustration provides a viable approach for industries seeking to adopt environmentally responsible inventory strategies without compromising operational goals. Future research may extend this framework to incorporate stochastic demand, carbon pricing mechanisms, and reverse logistics to further enhance the environmental integrity of industrial supply chains.

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