Earthline Journal of Mathematical Sciences
E-ISSN: 2581-8147
Volume 16, Number 3, May 2026, Pages 413-425
https://doi.org/10.34198/ejms.16326.30.413425

Mathematical Model of a Four Wheel Conventional Robot

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Abstract

Mobile robot creation, control, and optimization demand adaptive and efficient strategies. The motion of a four-wheel mobile robot is considered. The four wheels are organized in pairs, with each pair set up as a differential drive system. The robot's motion is analyzed at both kinematic and dynamic levels. Simulation tests validate the proposed algorithm, and the impact of the forces on the wheels is discussed, leading to key conclusions.

1 Introduction

The build of a mobile robot consists of the chassis of the robot and the wheels of the robot. The description of motion of the two-wheeled mobile robots is presented in [1]. In [2], the mathematical model for the design and construction of a 4-wheel mobile robot has been presented. They reported that the control structure provided the benefit of economical utilization of energy sources. Their simulated model calculated the intended path of the undercarriage center of gravity and the path of the wheels. A mathematical model of a 4-wheeled mobile robot with mecanum wheels was investigated in [3]. They used the Lagrange equations of the second kind in deriving the equations of motion. They reported on the accurateness of the numerical solution to the derived dynamic equations of motion. A new model for skid-steering mobile robots was discussed in [4]. They focused on the relationship between the frictional force and angular velocity of the mobile robot. They reported that the developed state-space feedback controller performed well. Mikova [5] focused on the undercarriage center of gravity and the path of the wheels. It was reported that the simulated experiments could be verified by experimental analysis. In [6], the position estimation of a mobile robot by sensing caster wheel motion was studied. The simulations were carried out using MATLAB. They reported that the angular velocity of the caster wheel and the heading angle calculated from the sensor output readings with the help of inverse kinematics equations matched well with that of actual values given as input for simulation. The mathematical model of a system containing mobile platforms for a mobile robot has been investigated in [7]. They reported that the developed model was suitable as a

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Received: November 10, 2025; Accepted: December 31, 2025; Published online: April 14, 2026
2020 Mathematics Subject Classification: 70Q05, 70E60.
Keywords and phrases: mobile robot, robot kinematics, robot dynamics, differential drive, robot wheels, simulation.

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control system. The posture stabilization of a humanoid upper-body robot was considered in [8]. The simulations were carried out using MATLAB. They reported that the robot was successful at navigating various obstacles at remarkably high speeds. In [9], mathematical models in robotics were studied. The study cut across soft continuum robots to autonomous mobile robots and artificial intelligence in robotics. It was reported that comparative insight is essential in model selection.In this work, the dynamics of motion of the mobile robot with four-wheel drive is considered, and the dynamic and kinematic models have been formulated. The simulation results have also been included in the further sections.

2 Mathematical Model

The build of the mobile robot consists of the chassis of the robot and the wheels of the robot. The robot operates on a horizontal plane having a total dimensionality of three; two for the position in the plane and one for orientation orthogonal to the plane. The chassis of the robot is assumed to be rigid. Essentially, it shall be modeled as a rigid body on wheels.

The wheels of the robot are four in total, organized in pairs: two on the right hand side and another two on the left hand side. The wheels drive in the same direction, which in turn enables the robot to drive and steer. This mechanism is similar to the 'differential drive' as it is based on the control of the relative velocities of the left and right wheels. The kinematic model as well as the dynamic model of the robot shall be presented.

2.1 Kinematic Model

The inertial frame, the body attached frame, and the wheel attached frame are utilized in specifying the position of the robot. Consequently, the coordinate system is such that the three coordinate frames and their corresponding axes are described thus: the inertial frame or space frame S with axis (x_s, y_s, z_s); the body attached frame B with origin at the centroid of the robot has axis (x_B, y_B, z_B); and the wheel attached frame E with axis (x_E, y_E, z_E). Frame S does not move. The only rotation of the robot's chassis with respect to frame S is rotation about z_s with angle θ. It is the angle of x_B with respect to x_s. x_E is perpendicular to the axis of the wheel. y_E is the axis of the rotation of the wheel. x_E is perpendicular to y_E such that z_E is in the same direction as z_B and z_s. See Figure 1 for a sketch of the model.

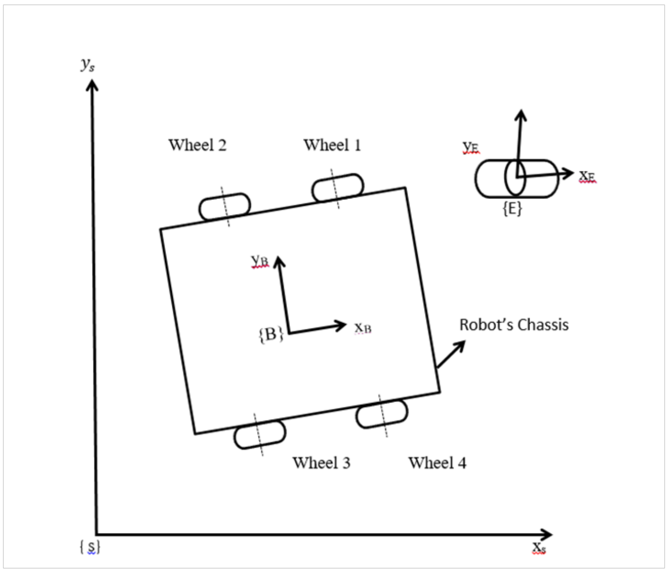

Figure 1: Sketch of mobile base with four conventional wheels indicating the three frames of reference.

Frame E is localized with origin (x_i, y_i) whereby the wheels are fixed and conventional; as a result, there is infinite resistance in the lateral direction. The direction of the motion for each wheel is given by ϕ_i, which is measured in the counter-clockwise direction with respect to y_E. x_E and y_B are parallel, and x_B is the direction of the motion of the robot.

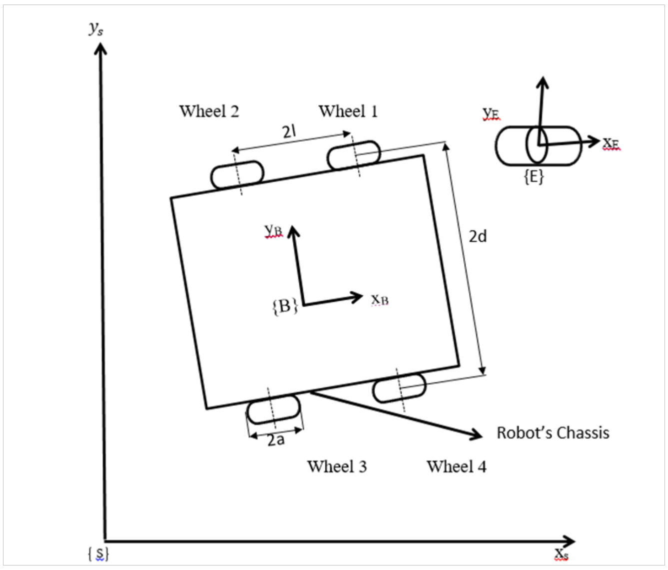

Figure 2: Dimensions of four wheel mobile robot.

The kinematic model of a mobile robot is generally given by:

$$\dot{\eta} = J(\psi)\,\zeta, \tag{1a}$$

where:

- $\dot{\eta}$ represents the time derivatives of the generalized coordinates,
- $J(\psi)$ is the Jacobian matrix that maps the input velocity commands to the derivatives of the generalized coordinates,
- ψ denotes the orientation variable,
- ζ represents the velocity input commands.

However, as a result of the wheel configuration, the kinematic model of the mobile base can be expressed as:

$$\zeta = W\,\omega, \tag{1b}$$

where:

- W is the wheel input (configuration) matrix,
- ω represents the wheel angular velocity vector.

The kinematic model of this mobile robot shall be developed using the wheel configuration model. The wheel configuration model gives the relationship between the wheel angular velocity and the velocity input commands.

Recall that, the equation of the generalized wheel is given below:

$$\omega_i = \begin{bmatrix} \frac{1}{a_i} & \frac{1}{a_i}\tan\phi_i \end{bmatrix} \begin{bmatrix} \cos\theta_{Bi} & \sin\theta_{Bi} \\ -\sin\theta_{Bi} & \cos\theta_{Bi} \end{bmatrix} \begin{bmatrix} 1 & 0 & -dy_i \\ 0 & 1 & dx_i \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{2}$$

where

- ω_i = angular velocity of the ith wheel,
- a_i = radius of the ith wheel,
- θ_{B_i} = angle between the body-attached frame B and the wheel-attached frame E_i,
- dx_i, dy_i = position coordinates of E_i with respect to frame B,
- ϕ_i = angle between the roller axis and the x_{E_i} axis,
- u = forward velocity of the mobile robot with respect to frame B,
- v = lateral velocity of the mobile robot with respect to frame B,

- r = angular velocity of the mobile robot with respect to frame B.

Note that the number of wheels of the mobile base is 4: $i = 1, 2, 3, 4$, where i is the number of wheels, and as further described in Figure 2, our mobile base is hinged on these conditions:

$$a_1 = a_2 = a_3 = a_4 = a, \quad \phi_1 = \phi_2 = \phi_3 = \phi_4 = 0,$$

$$\theta_{B1} = \theta_1 = 0, \quad \theta_{B2} = \theta_2 = 0, \quad \theta_{B3} = \theta_3 = 0, \quad \theta_{B4} = \theta_4 = 0,$$

$$dx_1 = l, \quad dy_1 = d, \quad dx_2 = -l, \quad dy_2 = d, \quad dx_3 = -l, \quad dy_3 = -d, \quad dx_4 = l, \quad dy_4 = -d \tag{3}$$

where, d = distance from frame E to frame B in the y-axis, l = distance from the centre of the wheel to the outer edge in the x-axis, and all other parameters retain their initial meaning.

Substituting equations (3) in equation (2) we have that:

For wheel 1:

$$\omega_1 = \begin{bmatrix} 1/a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \frac{1}{a}(u - dr) \tag{4a}$$

For wheel 2:

$$\omega_2 = \begin{bmatrix} 1/a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & -l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \frac{1}{a}(u - dr) \tag{4b}$$

For wheel 3:

$$\omega_3 = \begin{bmatrix} 1/a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & -l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \frac{1}{a}(u + dr) \tag{4c}$$

For wheel 4:

$$\omega_4 = \begin{bmatrix} 1/a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \frac{1}{a}(u + dr) \tag{4d}$$

Putting equations (4) in matrix form, we have that:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} 1/a & -d/a \\ 1/a & -d/a \\ 1/a & d/a \\ 1/a & d/a \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} \tag{5}$$

Equation (5) gives the generalized wheel model of a skid-steering mobile base. Comparing Equation (5) to Equation (1b), we will need to obtain the wheel inputs, after which we will multiply the wheel input by the wheel angular velocity in order to ultimately obtain the kinematic model of this 4-wheel mobile robot.

To get the wheel input, take the pseudo-inverse of the matrix multiplying the velocity input command matrix contained in Equation (5). Mathematically:

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{pmatrix} \frac{a}{4} & \frac{a}{4} & \frac{a}{4} & \frac{a}{4} \\ -\frac{a}{4d} & -\frac{a}{4d} & \frac{a}{4d} & \frac{a}{4d} \end{pmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \tag{6}$$

Here the wheel input W, is given by:

$$W = \begin{pmatrix} \frac{a}{4} & \frac{a}{4} & \frac{a}{4} & \frac{a}{4} \\ -\frac{a}{4d} & -\frac{a}{4d} & \frac{a}{4d} & \frac{a}{4d} \end{pmatrix} \tag{7}$$

Therefore:

$$\begin{bmatrix} u \\ v \\ r \end{bmatrix} = \zeta = \begin{pmatrix} \frac{a}{4} & \frac{a}{4} & \frac{a}{4} & \frac{a}{4} \\ 0 & 0 & 0 & 0 \\ -\frac{a}{4d} & -\frac{a}{4d} & \frac{a}{4d} & \frac{a}{4d} \end{pmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = W\omega \tag{8}$$

Equation (8) is the kinematic model based on wheel inputs configuration.

2.2 Dynamic Model

To deduce the dynamic model, we take note of the force-moment effects generated in frame E relative to frame B, all in the inertial frame. See Figure 3 for a sketch of the frame.

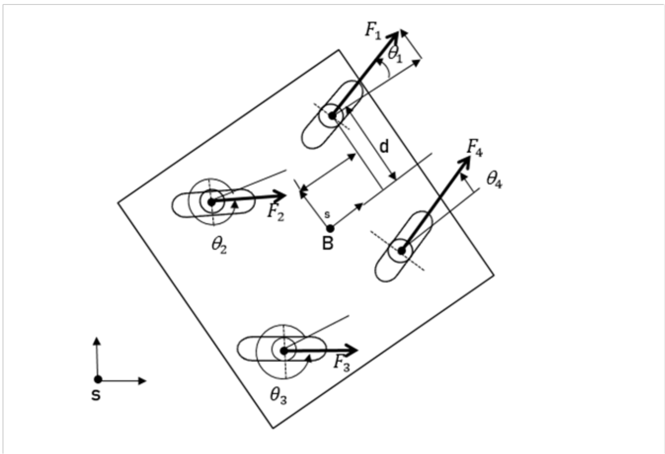

Figure 3: Force moment relationship on four wheel mobile robot.

Recall that the dynamic model of a mobile robot is given by:

$$D\dot{\zeta} + \eta(\zeta) = \tau, \tag{9}$$

where:

- D is the inertia matrix, such that $D^T = D > 0$,
- $\dot{\zeta}$ is the time derivative of the velocity inputs,
- ζ represents the velocity input commands,
- $\eta(\zeta)$ denotes the nonlinear effects,
- τ is the vector of applied forces and moments with respect to frame B.

Based on the wheel configuration, the dynamic model can also be expressed as:

$$\tau = \Gamma\kappa, \tag{10}$$

where:

- Γ is the wheel input matrix,
- κ is the configuration vector.

The force–moment relationships are given by:

$$\begin{aligned} F_x &= \sum_{i=1}^{4} F_i \cos\theta_i, \\ F_y &= \sum_{i=1}^{4} F_i \sin\theta_i, \\ M_z &= \sum_{i=1}^{4} (\mathbf{P}_i \times \mathbf{F}_i), \end{aligned} \tag{11}$$

where, F_x and F_y are forces relative to the x-axis and y-axis respectively,

- F_i = force component on the ith wheel,
- M_z = moment of force,
- $\mathbf{P}_i$ = position vector (of frame E relative to frame B),
- $\mathbf{F}_i$ = force on the ith wheel.

The position vectors are defined as:

$$\begin{aligned} \mathbf{P}_1 &= [l \quad d \quad 0]^T, \\ \mathbf{P}_2 &= [-l \quad d \quad 0]^T, \\ \mathbf{P}_3 &= [-l \quad -d \quad 0]^T, \\ \mathbf{P}_4 &= [l \quad -d \quad 0]^T. \end{aligned} \tag{12}$$

The corresponding force vectors are:

$$\begin{aligned} \mathbf{F}_1 &= [F_1 \cos\theta_1 \quad F_1 \sin\theta_1 \quad 0]^T, \\ \mathbf{F}_2 &= [F_2 \cos\theta_2 \quad F_2 \sin\theta_2 \quad 0]^T, \\ \mathbf{F}_3 &= [F_3 \cos\theta_3 \quad F_3 \sin\theta_3 \quad 0]^T, \\ \mathbf{F}_4 &= [F_4 \cos\theta_4 \quad F_4 \sin\theta_4 \quad 0]^T. \end{aligned} \tag{13}$$

Substituting these into the force–moment equations gives:

$$\begin{aligned} F_x &= F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4, \\ F_y &= F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4, \\ M_z &= lF_1 \sin\theta_1 - dF_1 \cos\theta_1 - lF_2 \sin\theta_2 - dF_2 \cos\theta_2 \\ &\quad + lF_3 \sin\theta_3 + dF_3 \cos\theta_3 + lF_4 \sin\theta_4 + dF_4 \cos\theta_4. \end{aligned}$$

The above equations can be written in matrix form as:

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \tau = \begin{bmatrix} \cos\theta_1 & \cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \sin\theta_1 & \sin\theta_2 & \sin\theta_3 & \sin\theta_4 \\ (l\sin\theta_1 - d\cos\theta_1) & -(l\sin\theta_2 + d\cos\theta_2) & (-l\sin\theta_3 + d\cos\theta_3) & (l\sin\theta_4 + d\cos\theta_4) \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \Gamma\kappa \tag{14}$$

Substituting the values of θ_i for $i = 1, 2, 3, 4$ from Equation (3), Equation (14) reduces to:

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \tau = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ -d & -d & d & d \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \Gamma\kappa \tag{15}$$

Equation (15) represents the required dynamic model.

3 Simulation Results

Having gotten the kinematic and dynamic models of a mobile base of four conventional wheels, we proceed to carry out the simulation of the models.

3.1 Result and Discussion

Numerical computations have been carried out using MATLAB, and results are presented graphically. The default values of the parameters throughout the simulation are considered as $d = 0.2$, $l = 0.6$, $w = 0.4$, $F_1 = 0.5$, $F_2 = 0.5$, $F_3 = 0.5$, $F_4 = 0.5$, the mass of the robot is 10, and the moment of inertia of the robot is 0.1.

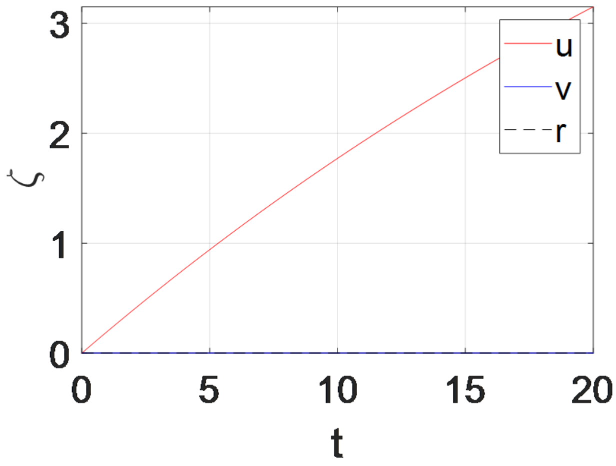

Figure 4: Velocity-time graph for equal forces acting on each of the wheel.

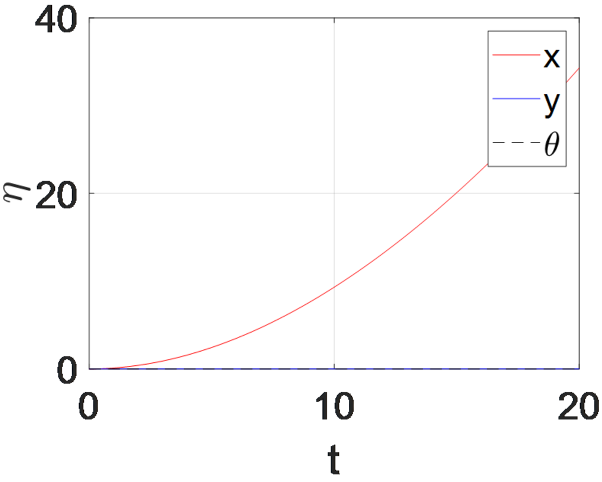

Figure 5: Position-time graph for equal forces acting on each of the wheel.

Figure 4 depicts the velocity of the mobile robot as time passes, with the forces on each of the four wheels being equal. It is observed that the robot's forward velocity increases as time passes. It is also observed that there is no change in the lateral velocity of the mobile robot, nor is there a change in the angular velocity of the mobile robot as time passes. And as such, the motion of the robot is straightforward only. It is worthy to note that the straight motion also holds for equal and opposite forces on each of the wheels, where the robot would move in the left direction, though straight. Plots of the generalized coordinates as time passes of a mobile robot having equal forces on each of the four wheels are presented in Figure 5. It can be deduced that the horizontal position of the robot increases from left to right as the robot is in motion. However, the vertical position as well as the orientation of the robot are unchanged as the robot is in motion.

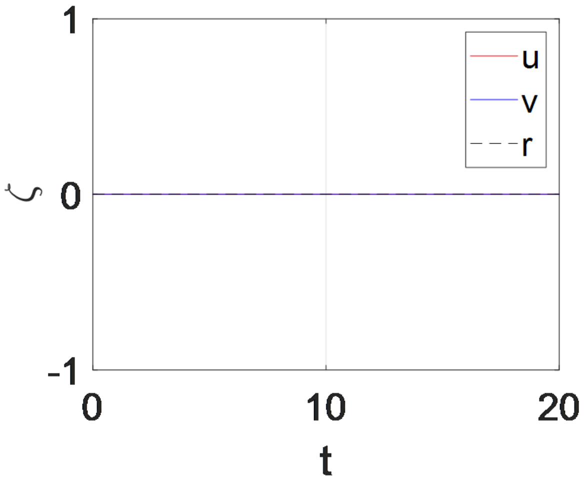

Figure 6: Velocity-time graph for forces acting on each wheel.

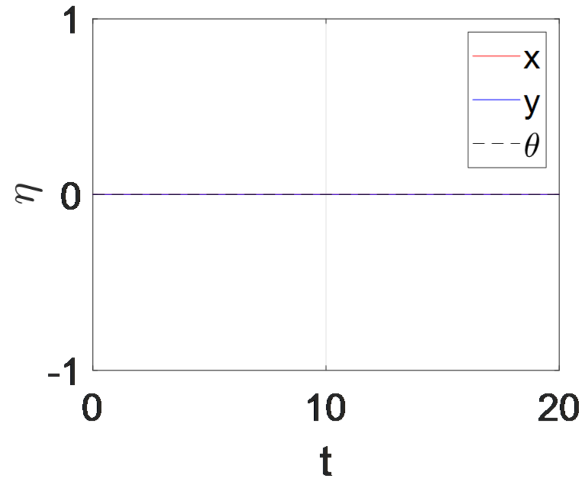

Figure 7: Position-time graph for forces acting on each wheel.

The force on the wheels captured in Figures 6 and 7 is given by the relationship $F_1 = F_4$, $F_2 = F_3$, but $F_2 = -F_1$. There is no forward or lateral velocity, nor is there any change in the angular velocity as time passes. Consequently, the robot is stationary.

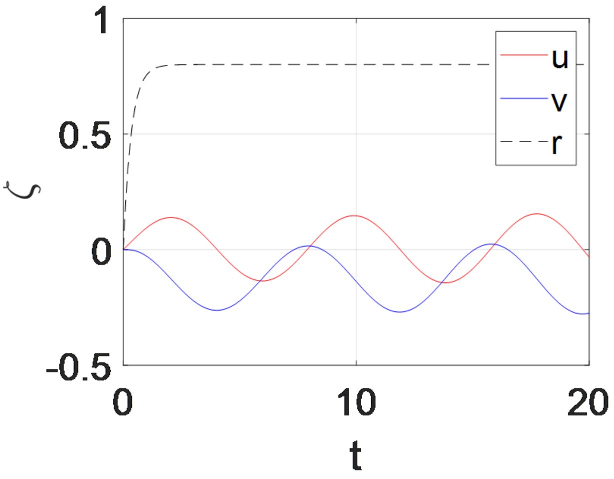

Figure 8: Velocity-time graph for different forces acting on each wheel.

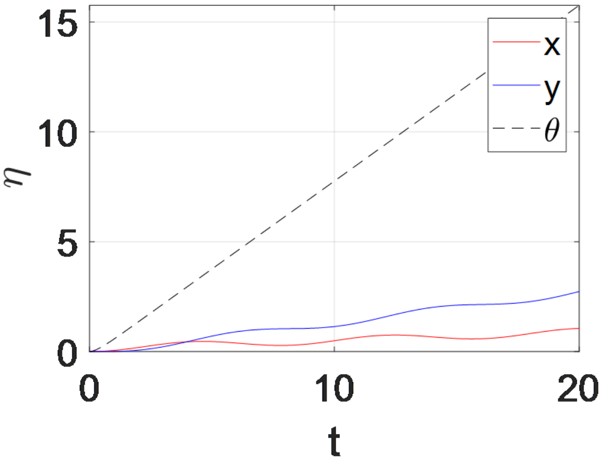

Figure 9: Position-time graph for different forces acting on each wheel.

The force acting on the wheels represented in Figures 8 and 9 is given by the relationship $F_1 = F_2$, $F_3 = F_4$, but $F_3 = -F_1$. There is no forward or lateral velocity. However, the robot rotates in space in a clockwise fashion. The motion of the robot is largely because the force on the right wheels is lowered, enabling the robot to turn to the right as the force on the left wheels is increased. For the robot to turn to the left, a similar situation is observed, with the force on the left wheels being decreased while the force on the right wheels is increased.

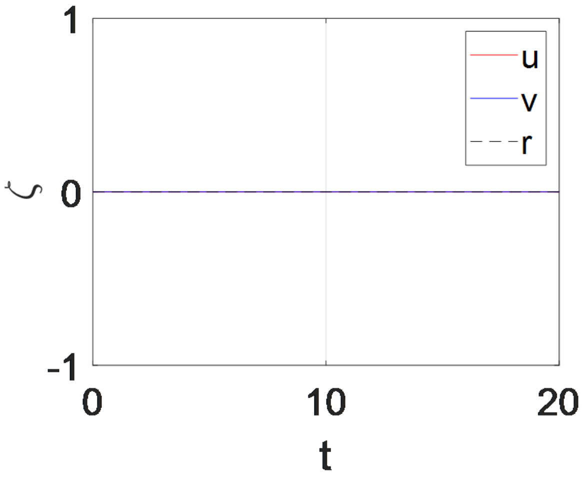

Figure 10: Velocity-time graph for null forces acting on each wheel.

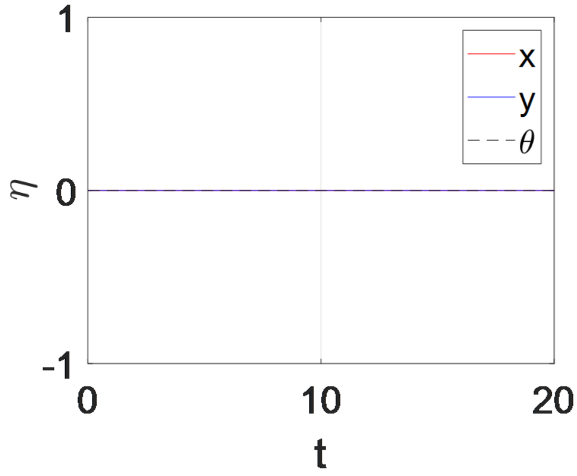

Figure 11: Position-time graph for null forces acting on each wheel.

For Figures 10 and 11, the force on the wheels is given by the relationship $F_1 = F_2 = F_3 = F_4 = 0$. When the forces on each wheel are null, the robot is stationary.

4 Conclusion

A mathematical model of a mobile robot of four conventional wheels with a chassis was considered. The kinematic and dynamic models were presented. Numerical simulation was conducted, and from the proposed model it was discovered that the mobile robot could successfully track the desired path.

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