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Corner Rules Method of Solving Transportation Problem

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Abstract

Several approaches have been advanced for solving transportation problems. The most prominent of them in various text being, North West Corner Rule(NWCR), Least Cost Method(LCM), and Vogel's Approximation Method(VAM). This paper considered three additional corner rules, which are North East Corner Rule(NECR), South West Corner Rule(SWCR) and South East Corner Rule(SECR). Algorithms ware provided for obtaining initial feasible solution to Transportation Problems. Three test examples were considered using the rules. The results revealed that the NECR and SWCR have equal result. While NWCR and SECR also produce the same result. NECR and SWCR however, better minimize transportation cost. The two methods are therefore recommended for use in any business organization requiring shipment of products.

Introduction

Transportation plays a very significant role on the cost of products and services. To maintain a reasonable standard of profitability level of a company, minimization of transportation cost is germane.

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Transportation problem(TP) as one of the earliest and most important approach of linear programming [6] deals with movement of goods and services from various sources(Origin) to several destinations at a minimum cost. It is an area of Operations Research prominently applied in production planning, communication network, inventory control scheduling as well as personal allocation. A Balanced Transportation Problem(BTP) is such that the total supply from all sources is equal to the total demand in all the destinations. Otherwise, the TP is called an Unbalanced Transportation Problem(UBTP).

Many researchers have proposed different methods of solving Transportation Problems. While reviewing literature on transportation problem, Anuradah et al. [1] noted the important role played by transportation problem in logistic and supply chain management towards cost minimization and effective service delivery. Sathyavathy and Shalini [8] approached solution of transportation problem using four different means: Arithmetic, Harmonic, Geometric and Quadratic mean, with optimal solutions. An alternative approach that finds the optimal or nearly optimal solution to the transportation problem, by Identifying the cell for allocation which has the least unit transportation cost (c_{ij}) in each row and columns was also proposed by Ekanayake et al. [3]. A Revised Distribution Method which involves maximizing objective function was proposed by Choudhary [2]. Sen et al. [7] obtained feasible solutions to transportation problems, by developing an object oriented programming. MATLAB programs were prepared for solving Transportation Problems via NWCR, LCM as well as the VAM. Mishra [9] compared solutions of Transportation problems obtained using North West Corner Rule, Minimum Cost and the Vogel's Approximation Method; with the observation that (VAM) is near optimal. Yadav et al. [11] compared Mean Proposed Methods with Minimum Value Methods of solving Transportation Problems, and observed that the Harmonic Mean Method(HMM) better minimizes transport cost. Hussein and Shiker [10] introduced a modification to the Vogel's Approximation Method for finding an Initial Basic Feasible Solution(IBFS) almost near to optimal solve.

Formulation of the Model

The transportation problem model is formulated in linear programming order as follows:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} Cijx_{ij}$
subject to: $\sum_{i=1}^{n} x_{ij} = s_i \; i = 1, 2, 3, 4, ...m$
$\sum_{i=1}^{n} x_{ij} = d_j \; j = 1, 2, 3, 4, ...n$
$x_{ij} \geq 0$

where

$x_{ij} =$ Quantity of goods shipped from i to j

$C_{ij} =$ Unit cost of transportation from warehouse/source i to market/destination j

$s_i =$ Number of supply points

$d_j =$ Number of demand points

$m =$ Number of available warehouse/source

$n =$ Number of available market/destination

The general transportation problem can be expressed in tabular order known as transportation array given in Table 1.

Table 1: Transportation array.

SOURCE/DESTINATION	$W1$	W_2	.....	W_n	SUPPLY
W_1	x_{11} $^{c_{11}}$	x_{12} $^{c_{12}}$	.....	x_{1n} $^{c_{1n}}$	s_1
W_2	x_{21} $^{c_{21}}$	x_{22} $^{c_{22}}$	.....	x_{2n} $^{c_{2n}}$	s_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
W_m	x_{m1} $^{c_{m1}}$	x_{m2} $^{c_{m2}}$	.....	x_{mn} $^{c_{mn}}$	s_m
DEMAND	d_1	d_2	.....	d_n	

Algorithm for Proposed Methods

This approach starts allocation of units to be transported to the cell corresponding to given corner of the rule, and proceed towards opposite corner direction following steps stated below:

Step 1: Select the cell of the transportation table that corresponds to; Upper Right-hand corner for NECR, Lower Right-hand corner for SECR and Lower Left-hand corner for SWCR and allocate as many units as possible equal to the minimum of the available supply and demand.

Step 2: Remain on the row or column until the supply or demand is exhausted.

Step 3: If both row and column are simultaneously exhausted, repeat the operation in Step 1. for the remaining rectangular transportation table.

Step 4: Continue the procedure until both demand and supply units are fully allocated.

Numerical Examples

Some numerical examples are considered to test the proposed methods.

Problem 1. Ajoke Investment is a major distributor for Dangote cement in Ilorin. It has three warehouses W_1, W_2, and W_3, where cement is stored for shipment to required market points. Three building sites B_1, B_2, and B_3 require the supply of cement from the company. The demand of the building site, the current capacity of each warehouse and the unit cost of transportation is as given in Table 2. Obtain a minimum cost of transportation.

Table 2: Transportation table.

SOURCE/DESTINATION	B_1	B_2	B_3	SUPPLY
W_1	^{40}	^{30}	^{50}	35
W_2	^{20}	^{60}	^{70}	43
W_3	^{30}	^{80}	^{90}	52
DEMAND	40	30	60	130

Table 3: Solution using North-East Corner Rule(NECR).

SOURCE/DESTINATION	B_1	B_2	B_3	SUPPLY
W_1	^{40}	^{30}	$35\ ^{50}$	35
W_2	^{20}	$18\ ^{60}$	$25\ ^{70}$	43
W_3	$40\ ^{30}$	$12\ ^{80}$	^{90}	52
DEMAND	40	30	60	130

$$Z = 50(35) + 70(25) + 60(18) + 80(12) + 30(40) = 6740$$

Table 4: Solution using South-East Corner Rule(SECR).

SOURCE/DESTINATION	B_1	B_2	B_3	SUPPLY
W_1	$35\ ^{40}$	^{30}	^{50}	35
W_2	$5\ ^{20}$	$30\ ^{60}$	$8\ ^{70}$	43
W_3	^{30}	^{80}	$52\ ^{90}$	52
DEMAND	40	30	60	130

$$Z = 90(52) + 70(8) + 60(30) + 20(5) + 40(35) = 8540$$

Table 5: Solution using South-West Corner Rule(SWCR).

SOURCE/DESTINATION	B_1	B_2	B_3	SUPPLY
W_1	40	30	35 50	35
W_2	20	18 60	25 70	43
W_3	40 30	12 80	90	52
DEMAND	40	30	60	130

$$Z = 50(35) + 70(25) + 60(18) + 80(12) + 30(40) = 6740$$

Solution using North-West Corner Rule(NWCR), Least Cost Method(LCM), and Vogel Approximation Method(VAM)

i. NWCR =8540

ii. LCM =6840

iii. VAM =6440

Problem 2. A special Bread baker has three distribution centers located at Amoyo(A), Bala(B), and Ganmo(G). This centers have available 60, 80 and 60 packs of special bread. His retail outlets in four markets M_1, M_2, M_3 and M_4 require 50, 65, 30 and 55 packs of the special bread respectively. The cost of transportation per unit in naira between each center and outlet is given in Table 6. Obtain a minimum cost of transportation.

Table 6: Transportation table.

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
A	5	8	7	6	60
B	10	9	11	12	80
G	6	5	13	10	60
DEMAND	50	65	30	55	200

Table 7: Solution using North-East Corner Rule(NECR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
A	5	8	5 7	55 6	60
B	10	55 9	25 11	12	80
G	50 6	10 5	13	10	60
DEMAND	50	65	30	55	200

$$Z = 6(55) + 7(5) + 11(25) + 9(55) + 5(10) + 6(50) = 1485$$

Table 8: Solution using South-East Corner Rule(SECR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
A	50 5	10 8	7	6	60
B	10	55 9	25 11	12	80
G	6	5	5 13	55 10	60
DEMAND	50	65	30	55	200

$$Z = 10(55) + 13(5) + 11(25) + 9(55) + 8(10) + 5(50) = 1715$$

Table 9: Solution using South-West Corner Rule(SWCR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
A	5	8	5 7	55 6	60
B	10	55 9	25 11	12	80
G	50 6	10 5	13	10	60
DEMAND	50	65	30	55	200

$$Z = 6(50) + 5(10) + 9(55) + 11(25) + 7(5) + 6(55) = 1485$$

Solution using North-West Corner Rule(NWCR), Least Cost Method(LCM), and Vogel Approximation Method(VAM)

i. NWCR =1715

ii. LCM =1545

iii. VAM =1485

Problem 3. An wholesale marketer of biscuit has four warehouses W_1, W_2 W_3, and W_4, where cartons of biscuit are stored for shipment to required market points. Four retailers M_1, M_2, M_3 and M_4 require the supply of Biscuit from the Marketer. The demand of the retailers, the current capacity of each warehouse and the unit cost of transportation is as given in Table 10. Obtain a minimum cost of transportation.

Minimize the cost of transportation in Table 10 below.

Table 10: Transportation table.

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
W_1	9	11	8	12	140
W_2	5	10	7	3	90
W_3	3	2	5	4	120
W_4	4	6	8	6	80
DEMAND	120	100	80	130	430

Table 11: Solution using North-East Corner Rule(NECR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
W_1	9	11	10 8	130 12	140
W_2	5	20 10	70 7	3	90
W_3	40 3	80 2	5	4	120
W_4	80 4	6	8	6	80
DEMAND	120	100	80	130	430

$$Z = 12(130) + 8(10) + 7(70) + 10(20) + 2(80) + 3(40) + 4(80) = 2930$$

Table 12: Solution using South-East Corner Rule(SECR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
W_1	120 9	20 11	8	12	140
W_2	5	80 10	10 7	3	90
W_3	3	2	70 5	50 4	120
W_4	4	6	8	80 6	80
DEMAND	120	100	80	130	430

$$Z = 6(80) + 4(50) + 5(70) + 7(10) + 10(80) + 11(20) + 9(120) = 3200$$

Table 13: Solution using South-West Corner Rule(SWCR).

SOURCE/DESTINATION	M_1	M_2	M_3	M_4	SUPPLY
W_1	9	11	10 8	130 12	140
W_2	5	20 10	70 7	3	90
W_3	40 3	80 2	5	4	120
W_4	80 4	6	8	6	80
DEMAND	120	100	80	130	430

$$Z = 4(80) + 3(40) + 2(80) + 10(20) + 7(70) + 8(10) + 12(130) = 2930$$

Solution using North-West Corner Rule(NWCR), Least Cost Method(LCM), and Vogel Approximation Method(VAM)

i. NWCR =3200

ii. LCM =2150

iii. VAM =2170

Table 14: Table of result presentation.

PROBLEM/METHOD	NWCR	NECR	SWCR	SECR	LCM	VAM
1	8540	6740	6740	8504	6840	6440
2	1715	1485	1485	1715	1545	1485
3	3200	2930	2930	3200	2150	2170

Discussion of Results

The analysis of the Corner rule methods as shown in Table 12 indicates a pare similarities in the Initial Basic Feasible Solutions obtained using the NECR and

SWCR on one side, and NWCR and SECR on the other. The algorithms for the NECR and SWCR however produced equal and better solution than the others. It is evidence from the test examples that both NECR and SWCR compared favorably with the Vogel's Approximation Method believed to be the best cost minimization approach in transportation problems, with NWCR and SECR giving the worst result. This result will enhance the ability of business organizations to maximize their profit through minimization of the transportation cost. The proposed method has the limitations of not producing better and optimal solution compared with the Vogel's Approximation method.

Conclusion

In this paper, it is imperative to note that adoption of the best of the rules in logistics and supply chain decision making will go a long way in solving transportation problems of business organizations. The proposed algorithms are user friendly. This method can be used for both balanced and unbalanced transportation problems within a limited time frame compared to any other cost minimization method. Hence the method is highly recommended for inclusion in the list of the existing methods.

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