Earthline Journal of Mathematical Sciences
E-ISSN: 2581-8147
Volume 10, Number 2, 2022, Pages 409-421
https://doi.org/10.34198/ejms.10222.409421

Results of Semigroup of Linear Operator Generating a Quasilinear Equations of Evolution

J. B. Omosowon[1], A. Y. Akinyele[1,*],
B. M. Ahmed[1] and O. Y. Saka-Balogun[2]

[1] Department of Mathematics, University of Ilorin, Ilorin, Nigeria
e-mail: jbo0011@mix.wvu.edu
e-mail: olaakinyele04@gmail.com
e-mail: ahmed.bm@unilorin.edu.ng

[2] Department of Mathematical and Physical Sciences, Afe Babalola University, Ado-Ekiti, Nigeria
e-mail: balogunld@yahoo.com

Abstract

In this paper, results of ω-order preserving partial contraction mapping generating a quasilinear equation of evolution were presented. In general, the study of quasilinear initial value problems is quite complicated. For the sake of simplicity we restricted this study to the mild solution of the initial value problem of a quasilinear equation of evolution. We show that if the problem has a unique mild solution $v \in C([0,T]:X)$ for every given $u \in C([0,T]:X)$, then it defines a mapping $u \to v = F(u)$ of $C([0,T]:X)$ into itself. We also show that under the suitable condition, there exists always a T', $0 < T' \leq T$ such that the restriction of the mapping F to $C([0,T']:X)$ is a contraction which maps some ball of $C([0,t']:X)$ into itself by proving the existence of a local mild solution of the initial value problem.

Received: July 14, 2022; Accepted: August 16, 2022
2020 Mathematics Subject Classification: 06F15, 06F05, 20M05.
Keywords and phrases: ω-OCP_n, mild solution, C_0-semigroup, evolution equation.
*Corresponding author

1 Introduction

Consider the Cauchy problem for the quasilinear initial value problem

$$\begin{cases} \frac{du(t)}{dt} + A(t,u)u = 0 \quad for \quad 0 \leq t \leq T \\ u(0) = u_0 \end{cases} \tag{1.1}$$

in a Banach space X. In general, the study of quasilinear initial value problem is quite complex. For the sake of simplicity, we restricted this research to the mild solutions of the initial value problem of (1.1). Also, let $u \in C([0,T] : X)$ and consider the linear initial value problem

$$\begin{cases} \frac{dv}{dt} + A(t,u)v = 0 \quad for \quad 0 \leq t \leq T \\ v(0) = u_0. \end{cases} \tag{1.2}$$

If this problem has a unique mild solution $v \in C([0,T] : X)$, for every given $u \in C([0,T] : X)$, then it defines a mapping $u \to v = F(u)$ at $C([0,t] : X)$, for every given fixed points of this mapping are defined to be mild solution of (1.1). Suppose X is a Banach space, $X_n \subseteq X$ is a finite set, $\omega - OCP_n$ the ω-order preserving partial contraction mapping, M_m be a matrix, $L(X)$ be a bounded linear operator on X, P_n a partial transformation semigroup, $\rho(A)$ a resolvent set, $\sigma(A)$ a spectrum of A and A is a generator of C_0-semigroup. This paper consist of results of ω-order preserving partial contraction mapping generating a quasilinear equations of evolution. Agmon *et al.* [1], approximated some boundary problems for solutions of elliptic partial differential equation. Akinyele *et al.* [2], showed some perturbation results of the infinitesimal generator in the semigroup of the linear operator. Balakrishnan [3], presented an operator calculus for infinitesimal generators of semigroup. Banach [4], established and introduced the concept of Banach spaces. Batty *et al.* [5], proved some asymptotic behavior of semigroup of operators. Brezis and Gallouet [6], obtained nonlinear Schrodinger evolution equation. Chill and Tomilov [7], deduced some resolvent approach to stability operator semigroup. Davies [8], introduced linear operators and their spectra. Engel and Nagel [9], presented one-parameter semigroup for linear evolution equations. Omosowon *et al.* [10], proved some analytic results of semigroup of linear operator with dynamic boundary conditions, and also in [11], Omosowon *et al.*, established dual Properties of ω-order Reversing Partial Contraction Mapping in Semigroup of Linear Operator. Omosowon *et al.* [12], generated a regular weak*-continuous semigroup of linear operators. Pazy [13], introduced asymptotic behavior of the solution of an abstract evolution and some applications and also in [14], established a class of semi-linear equations of evolution. Prüss [15], proves some semilinear evolution equations in Banach spaces. Rauf and Akinyele [16], obtained ω-order preserving partial contraction mapping and established its properties, also in [17], Rauf *et al.*, introduced some results of stability and spectra properties on semigroup of linear operator. Vrabie [18], proved some results of C_0-semigroup and its applications. Yosida [19], established some results on differentiability and representation of one-parameter semigroup of linear operators.

2 Preliminaries

Definition 2.1 (C_0-semigroup) [18]
A C_0-semigroup is a strongly continuous one parameter semigroup of bounded linear operator on Banach space.

Definition 2.2 (ω-OCP_n) [16]
A transformation $\alpha \in P_n$ is called ω-order preserving partial contraction mapping if $\forall x, y \in$ Dom$\alpha : x \leq y \implies \alpha x \leq \alpha y$ and at least one of its transformation must satisfy $\alpha y = y$ such that $T(t+s) = T(t)T(s)$ whenever $t, s > 0$ and otherwise for $T(0) = I$.

Definition 2.3 (Evolution Equation) [20]
An evolution equation is an equation that can be interpreted as the differential law of the development (evolution) in time of a system. The class of evolution equations includes, first of all, ordinary differential equations and systems of the form

$$u' = f(t, u), u'' = f(t, u, u'),$$

etc., in the case where $u(t)$ can be regarded naturally as the solution of the Cauchy problem; these equations describe the evolution of systems with finitely many degrees of freedom.

Definition 2.4 (Mild Solution) [13]
A continuous solution u of the integral equation.

$$u(t) = T(t - t_0)u_0 + \int_{t_0}^{t} T(t - s)f(s, u(s))ds \tag{2.1}$$

will be called a mild solution of the initial value problem

$$\begin{cases} \frac{du(t)}{dt} + Au(t) = f(t, u(t)), \; t > t_0 \\ u(t_0) = u_0 \end{cases} \tag{2.2}$$

if the solution is a Lipschitz continuous function.

Definition 2.5 (Stable Family of Operator) [13]
Let B be a subset of X and for every $0 \leq t \leq T$ and $b \in B$, let $A(s, b)$ be the infinitesimal generator of a C_0-semigroup $T_{s,b}(t)t \geq 0$, on X. The family of operators $\{A(s, b)\}, \; (s, b) \in [0, T]XB$, is stable if there are constants $M \geq 1$ and w such that

$$\rho(A(s, b)) \supset (w, \infty) \quad \text{for} \quad (s, b) \in [0, T]XB \tag{2.3}$$

and

$$\left\| \prod_{j=1}^{k} R(\lambda : A(s_j, b_j)) \right\| \leq M(\lambda - w)^{-k} \quad \text{for} \quad \lambda > w \tag{2.4}$$

and every finite sequences $0 \leq t_1 \leq t_2 \leq \cdots \leq t_k \leq T$, $b_j \in B$, $1 \leq j \leq k$.

Assumption (F) 2.6: Let X and Y be Banach spaces such that Y is densely and continuously embedded in X. Let $B \subset X$ be a subset of X such that for every $(s, b) \in [0, T]XB$, $A(s, b)$ is the infinitesimal generator of C_0-semigroup $T_{s,b}(t, \; t \geq 0)$, on X. We then make the following assumptions:

(F_1) The family $\{A(s, b)\}$, $(s, b) \in [0, T]XB$ is stable.

(F_2) Y is $A(s,b)$-admissible for $(s,b) \in [0,T]XB$ and the family $\{A(s,b)\}(s,b) \in [0,T]XB$ of parts $\overline{A}(s,b)$ of $A(s,b)$ in Y, is stable in Y.

(F_3) For $(s,b) \in [0,t]XB$, $D(A(s,b)) \supset Y$, $A(s,b)$ is bounded linear operator from Y to X and $t \to A(s,b)$ is continuous in the $B(Y,X)$ norm $\|\|_{y\to X}$ for every $b \in B$.

(F_4) There is a constant L such that

$$\|A(s,b_1) - A(s,b_2)\|_{Y\to X} \leq \|b_1 - b_2\| \tag{2.5}$$

holds for every $b_1, b_2 \in B$ and $0 \leq t \leq T$.

(F_5) For every $u \in C([0,T]:X)$ satisfying $u(s) \in B$ for $0 \leq s \leq T$, we have

$$U_u(s,t)Y \subset Y, \quad 0 \leq t \leq s \leq T \tag{2.6}$$

and $U_u(s,t)$ is strongly continuous in Y for $0 \leq t \leq s \leq T$.

(F_6) Closed convex bounded subsets of Y are also closed in X.

Example 1

2×2 matrix $[M_m(\mathbb{N} \cup \{0\})]$

Suppose

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA}$, then

$$e^{tA} = \begin{pmatrix} e^{2t} & e^{I} \\ e^{t} & e^{2t} \end{pmatrix}.$$

Example 2

3×3 matrix $[M_m(\mathbb{N} \cup \{0\})]$

Suppose

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 2 & 2 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA}$, then

$$e^{tA} = \begin{pmatrix} e^{2t} & e^{2t} & e^{3t} \\ e^{2t} & e^{2t} & e^{2t} \\ e^{t} & e^{2t} & e^{2t} \end{pmatrix}.$$

Example 3

3×3 matrix $[M_m(\mathbb{C})]$, we have
for each $\lambda > 0$ such that $\lambda \in \rho(A)$ where $\rho(A)$ is a resolvent set on X.
Suppose we have

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 2 & 2 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA_\lambda}$, then

$$e^{tA_\lambda} = \begin{pmatrix} e^{2t\lambda} & e^{2t\lambda} & e^{3t\lambda} \\ e^{2t\lambda} & e^{2t\lambda} & e^{2t\lambda} \\ e^{t\lambda} & e^{2t\lambda} & e^{2t\lambda} \end{pmatrix}.$$

3 Main Results

This section present results of semigroup of linear operator by using ω-OCP_n to generates a quasilinear equations of evolution:

Theorem 3.1

Suppose $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a C_0-semigroup $\{T(t);\ t \geq 0\}$. Let $B \subset X$ and let $u \in C([0,T] : X)$ have values in B. If $\{A(s,b)\}$, $(s,b) \in [0,T]XB$ is a family of operators satisfying the assumptions $(F_1) - (F_4)$ such that $A \in \omega - OCP_n$. Then $\{A(s,u(s))\}_{s\in[0,T]}$ is a family of operators satisfying the assumptions $(F_1) - (F_3)$.

Proof:

From (F_1) and (F_2) it follows readily that $\{A(s,u(s))\}_t \in [0,T]$ satisfies (F_1) and (F_2). Moreover, it is clear from (F_3) that for $s \in [0,T]$, $D(A(s))$; $u(s) \supset Y$ and that $A(s,u(s))$ is a bounded linear operator from Y to X. It remains only to show that $s \to A(s,u(s))$ is continuous in the $B(Y,X)$ norm. But by (F_4), we have

$$\begin{aligned} &\|A(s_1,u(s_1)) - A(s_2,u(s_2))\|_{Y-X} \\ &\qquad \le \|A(u_1,u(s_1)) - A(s_2,u(s_1))\|_{Y-X} + C\|u(s_1) - u(s_2)\|. \quad (3.1) \end{aligned}$$

Since $u(s)$ is continuous in X, then the continuity of $s \to A(s,u(s))$ together with (3.1) imply that continuity of $s \to A(s,u(s))$ in the $B(Y,X)$ norm and this achieved the proof.

Theorem 3.2

Assume $A : D(A) \subseteq X \to X$ is the infinitesimal generator of C_0-semigroup $\{T(t);\ t \ge 0\}$. Let $B \subset X$ and let $\{A(s,b)\}$, $(s,b) \in [0,T]XB$, satisfying the conditions $(F_1)-(F_4)$ such that $A \in \omega - OCP_n$. Then there is a constant C_1 such that for every $u,v \in C([0,T] : X)$ with values in B and every $w \in Y$ and we have

$$\|U_u(s,t)w - U_u(s,t)w\| \le C\|w\|_Y \int_t^s \|A(\eta,u(\eta) - v(\eta)\|d\eta. \quad (3.2)$$

Proof:

Suppose

$$\|U_u(s,t)\| \le Me^{w(s-t)} \quad \text{for} \quad 0 \le t \le s \le T \quad (3.3)$$

and

$$\left.\frac{\partial^+}{\partial s}U_u(s,t)w\right|_{s=t} = A(t,u(t))w \quad (3.4)$$

so that

$$\frac{\partial}{\partial t}U_u(s,t)w = -U_u(s,t)A(t,u(t))w \quad (3.5)$$

for $w \in Y$, $A \in \omega - OCP_n$ and $0 \le t \le s \le T$. We have for every function $u \in C([0,t] : X)$ with values in B and $u_0 \in X$ the function $v(s) = U_u(s,0)_{u_0}$ is define to be the mild solution of the initial value problem (1.2). Since the semigroup satisfies the assumptions $(F_1) - (F_4)$ and if $u \in C([0,T] : X)$ has values in B then there is a unique evolution system $U_u(s,t)$, $0 \le t \le s \le T$ in X satisfying (3.3), (3.4) and (3.5). Then for every $u_0 \in X$ and $u \in C([0,T] : X)$ with values in B then the initial value problem (1.2) possesses a unique mild solution v given by

$$v(s) = U_u(s, 0)u_0. \tag{3.6}$$

Then we have

$$\|U_u(s,t)w - U_u(s,t)w\| \leq C\|w\|_Y \int_t^s \|A(\eta, u(\eta)) - A(\eta, v(\eta))\|_{Y \to X} d\eta, \tag{3.7}$$

where C depends only on the stability constants (3.7) of $\{A(s,b)\}$ and $\{\overline{A}(s,b)\}$. Combining (3.7) with (F_4) yields (3.2) and this completes the proof.

Theorem 3.3

Suppose $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a C_0-semigroup $\{T(t);\ t \geq 0\}$. Let $u_0 \in Y$ and let $B = \{x : \|x - u_0\| \leq r\}$, $r > 0$. If $\{A(s,b)\}$, $(s,b) \in [0,T]XB$ satifying the assumptions $(F_1) - (F_4)$, then there is a T', $0 < T' \leq T$ such that the initial value problem

$$\begin{cases} \frac{du}{ds} + A(s,u)u = 0, & 0 \leq s \leq T' \\ u(0) = u_0 \end{cases} \tag{3.8}$$

has a unique mild solution $u \in C([0,T'] : X)$ with $u(s) \in B$ and $A \in \omega - OCP_n$ for $0 \leq s \leq T'$.

Proof:

We note first that the constant function $u(s) \equiv u_0$ satisfies the assumptions of Theorem 3.2 and there is therefore an evolution system $U_{u_0}(s,t)$, $0 \leq t \leq s \leq T$ associated to u_0. Let $0 < t_1 \leq T$ be such that

$$\max_{0 \leq s \leq s_1} \|U_{u_0}(s, 0)u_0 - u_0\| < \frac{r}{2}$$

and choose

$$T' = \min\left\{s_1,\ \frac{1}{2}(C\|u_0\|_Y + 1)\right\} \tag{3.9}$$

where C is the constant appearing in Theorem 3.2. On the closed subset ξ of $C([0,T'] : X)$ defined by

$$\xi = \{u : u \in C([0,T'] : X),\ u(0) = u_0,\ \|u(s) - u_0\| \leq r \quad for \quad 0 \leq s \leq T\}, \tag{3.10}$$

we can consider the mapping

$$Fu(s) = U_u(s,0)u_0 \quad for \quad 0 \le s \le T'. \tag{3.11}$$

By our assumptions and Theorem 3.2, it is clear that F is well defined on ξ and that its range is in $C([0,T'] : X)$. We claim that $F : \xi \to \xi$. Indeed, for $u \in \xi$ we clearly have $Fu(0) = u_0$ and Theorem 3.2 and (3.9), we have

$$\begin{aligned} \|Fu(s) - u_0\| &\le \|U_u(s,0)u_0 - U_{u_0}(s,0)u_0\| + \|U_{u_0}(s,0)u_0 - u_0\| \\ &\le Cr\|u_0\|_Y T' + \frac{r}{2} \le r. \end{aligned}$$

Moreover, if $u_1, u_2 \in \xi$, then by Theorem 3.2 we have

$$\begin{aligned} \|Fu_1(s) - Fu_2(s)\| &= \|U_{u_1}(s,0)u_0 - U_{u_2}(s,0)u_0\| \\ &\le C\|u_0\|_Y \int_0^s \|u_1(\eta) - u_2(\eta)\| d\eta \\ &\le C\|u_0\|_Y T' \|u_1 - u_2\|_\infty \\ &\le \frac{1}{2}\|u_1 - u_2\|_\infty \end{aligned} \tag{3.12}$$

where $\| \ \|_\infty$ is the usual supremum norm in $C([0,T'] : X)$. From (3.12), it follows readily that

$$\|Fu_1 - Fu_2\|_\infty \le \frac{1}{2}\|u_1 - u_2\|_\infty \tag{3.13}$$

so that F is a contraction. From the contraction mapping theorem it follows that F has a unique fixed point $u \in \xi$ which is the desired mild solution of (3.8) on $[0,T']$. Hence the proof is completed.

Theorem 3.4

Assume $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a C_0-semigroup $\{T(t);\ t \ge 0\}$. Let $u_0 \in Y$ and let $B = \{y : \|y - u_0\|_Y \le r\}$, $r > 0$. Let $\{A(s,b)\}$, $(s,b) \in [0,T]XB$ be a family of linear operators satisfying the assumptions $(F_1) - (F_6)$. If $A(s,b)u_0 \in Y$, $A \in \omega - OCP_n$ and

$$\|A(s,b)u_0\|_Y \le k \quad for \quad (s,b) \in [0,T]XB \tag{3.14}$$

then there exists a T', $0 < T' \leq T$ *such that the initial value problem* (3.8) *has a unique classical solution* $u \in C([0, T'] : B) \cap C'([0, T] : X)$.

Proof:

We start by showing the existence of a unique nild solution of (3.8). We also note first that from the construction of $U_u(s, t)$ and (F_5), it then follows that

$$\|U_u(s, t)\|_Y \leq C_1 \quad for \quad 0 \leq t \leq s \leq T \tag{3.15}$$

and every $u \in C([0, T'] : X)$ with values in B. After choosing

$$T' = \min\left\{T,\ \frac{1}{KC_1},\ \frac{1}{2}(C\|u_0\|_Y + 1)^{-1}\right\} \tag{3.16}$$

where C is the constant appearing in Theorem 3.2, we consider the subset ξ of $C([0, T'] : X)$ defined by

$$\xi = \{u : u \in C([0, T'] : X),\ u(0) = u_0,\ u(s) \in B \quad for \quad 0 \leq s \leq T'\}.$$

From (F_6) it follows that ξ is a closed convex subset of $C([0, T] : X)$. Next we defined on ξ the mapping

$$Fu(s) = U_u(s, 0)u_0 \quad 0 \leq s \leq T' \tag{3.17}$$

and shows that $F : \xi \to \xi$. Clearly, $Fu(0) = u_0$ and $Fu(t) \in C([0, T] : X)$. From (F_5) it follows that $Fu(t) \in Y$ for $0 \leq S \leq T'$ and it remains to show that $\|Fu(s) - u_0\|_Y \leq r$ for $0 \leq s \leq T'$. Integrating (3.5) in X from t to s we find

$$U_u(s, 0)u_0 - u_0 = \int_0^s U_u(s, \eta)A(\eta, u(\eta))u_0 d\eta. \tag{3.18}$$

Estimating (3.18) in Y and using (3.14), (3.15) and (3.16) yields

$$\|Fu(s) - u_0\|_Y = \|U_u(s, 0)u_0 - u_0\|_Y \leq C_1 kT \leq r.$$

Therefore, $F : \xi \to \xi$. Now exactly as in the proof of Theorem 3.3, we also have for any $u_1, u_2 \in \xi$ so that

$$\|Fu_1 - Fu_2\|_\infty \leq \frac{1}{2}\|u_1 - u_2\|_\infty$$

where $\|\ \ \|$ is the supremum norm in $C([0,T']:X)$. Thus by the contraction theorem, F has a unique fixed point $u \in \xi$ which is the mild solution of (3.8) of $[0,T']$. But $u(s) = u_u(s,0)_{u_0}$ and therefore by (F_5) and Theorem 3.2, u is the unique Y-valued solution of the linear evolution equation

$$\begin{cases} \frac{dv}{ds} + A(s,u)v = 0 \\ v(0) = u_0 \end{cases} \tag{3.19}$$

and thus u is a classical solution of (3.8) and $u \in C([0,T]:Y) \cap C'([0,T]:X)$. The uniqueness of u is obvious and proof is complete.

Conclusion

In this paper, it has been established that ω-order preserving partial contraction mapping generates some results of quasilinear equations of evolution.

Acknowledgment

We acknowledge the management of the University of Ilorin for providing us with a suitable research laboratory and library to enable us carried out this research

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